Substitution and Integration by Parts (Again)

(covers parts of Stewart 5.5 and 7.1)

Substitution

When using the substitution method with definite integrals, change your limits to be in terms of u.

Ex 1.

Evaluate:

$$\int_{x=-1}^{x=1} 3x^2\sqrt{x^3+1}\,dx$$

$u = x^3+1$

$du = 3x^2\,dx$

$x=-1$: $u = (-1)^3+1 = 0$

$x=1$: $u = (1)^3+1 = 2$

$$= \int_{u=0}^{u=2} \sqrt{u}\,du$$

$$= \int_0^2 u^{1/2}\,du$$

$$= \frac{2}{3}u^{3/2}\Big]_0^2$$

$$= \frac{2}{3}(2)^{3/2} - \frac{2}{3}(0)^{3/2}$$

$$= \boxed{\frac{4\sqrt{2}}{3}}$$

Ex 2.

Evaluate:

$$\int_{\pi/4}^{\pi/2} \cot x \csc^2 x\,dx$$

$u = \cot x$

$du = -\csc^2 x\,dx$

$-du = \csc^2 x\,dx$

$x = \frac{\pi}{4}$: $u = \cot\frac{\pi}{4} = 1$

$x = \frac{\pi}{2}$: $u = \cot\frac{\pi}{2} = 0$

$$= \int_1^0 u(-du)$$

$$= \frac{-u^2}{2}\Big]_1^0$$

$$= \frac{u^2}{2}\Big]_0^1$$

$$= \frac{1^2}{2} - \frac{0^2}{2}$$

$$= \boxed{\frac{1}{2}}$$

Ex 3.

Evaluate:

$$\int_{-\pi/4}^{\pi/4} \tan x \, dx$$

Integration by Parts

Ex 4.

$$\int_0^1 xe^x \, dx$$

Ex 5.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \sin x \ln \sin x \, dx$$

Recall that $f(x)$ is even if $f(-x) = f(x)$, and $f(x)$ is odd if $f(-x) = -f(x)$.
If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.
If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0$.

Ex 6.
Evaluate:
$$\int_{-2}^{2} (x^4 - 4x^2 + 6)\,dx$$

Ex 7.
Evaluate:
$$\int_{-\pi}^{\pi} \sin x\,dx$$