Due date: _____ **Name:** _____

Tools for Integration (Part 1)

Notes

Sequences are lists of numbers. For example: $1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots$

We can generate a sequence with a formula. For example: $a_n = \frac{n}{2n-1}$ generates the above sequence.

ex: Find the first 4 terms and the 100th term of $a_n = \frac{(-1)^n}{n^2}$.

1. Find the first 4 terms of the following sequences.

$a_n = 5n$: _____

$a_n = 5n + 1$: _____

$a_n = 5n - 3$: _____

What do you notice about the patterns made by each sequence? _____

2. Find the first 6 terms of $a_n = \frac{3n}{2^n}$

What pattern does the 2^n make? _____

3. Find the first 4 terms and the 100th term of $a_n = \frac{(-1)^n}{4n-3}$.

What pattern does the $(-1)^n$ make? _____

Notes

A **series** is the sum of the terms of a sequence. For example: $1+\frac{2}{3}+\frac{3}{5}+\frac{4}{7}+\cdots$

We can use **sigma notation** to write a series. For example: $\sum_{n=1}^{5}\frac{n}{2n-1}$ is the sum of the first 5 terms of $\frac{n}{2n-1}$. That is, $\sum_{n=1}^{5}\frac{n}{2n-1}=1+\frac{2}{3}+\frac{3}{5}+\frac{4}{7}+\frac{5}{8}=\frac{2909}{840}$.

ex: Evaluate $\sum_{n=1}^{4}2^{n-1}$.

You can use a different **index**. For example: $\sum_{i=1}^{4}2^{i-1}$ is the same as $\sum_{n=1}^{4}2^{n-1}$.

4. Evaluate $\sum_{n=1}^{5}(2n-1)$

5. Evaluate $\sum_{i=0}^{3}\frac{2^i}{i+1}$

Notes

Here are some important summations to know:

$$\sum_{i=1}^{n} c = nc \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

ex: Evaluate $\sum_{i=1}^{25} 3$.

ex: Evaluate $\sum_{i=1}^{100} i$.

Also,

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = c \cdot \sum_{i=1}^{n} a_i$$

ex: Evaluate $\sum_{i=1}^{4} (5i^2 - 2i^3)$.

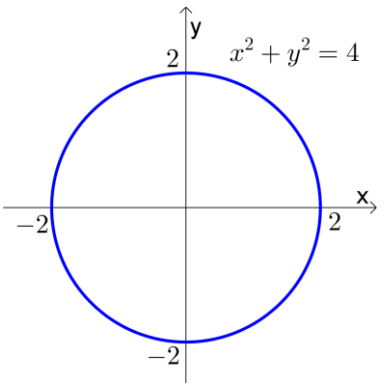
6. Use the summation formulas above to evaluate $\sum_{i=1}^{5} (2i + 3i^2 + 6)$

Notes

ex: Simplify $\sum_{i=1}^{n}\left(\frac{8}{n^3}i^3+\frac{2}{n}i\right)\frac{2}{n}$

7. Simplify $\sum_{i=1}^{n}\left(\frac{6}{n^2}i^2-4\right)\frac{2}{n}$

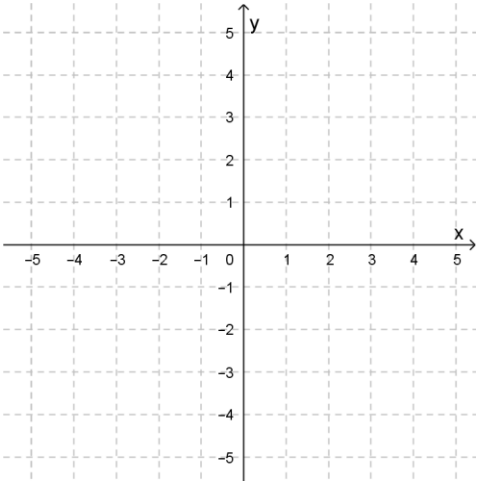
Notes



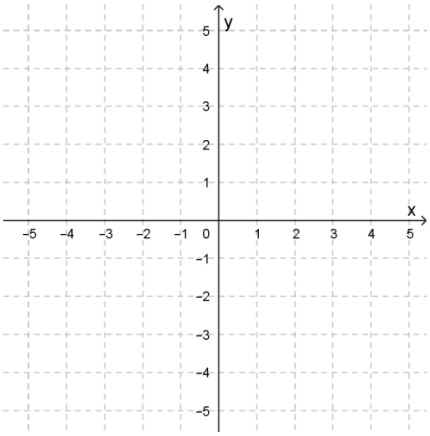
Recall the graph of $x^2 + y^2 = r^2$ is a circle centered at the origin with radius r. If you solve for y, you get:
$y = \pm\sqrt{r^2 - x^2}$

$y = \sqrt{r^2 - x^2}$ is the top half of the circle (with positive y-values).
$y = -\sqrt{r^2 - x^2}$ is the bottom half of the circle (with negative y-values).

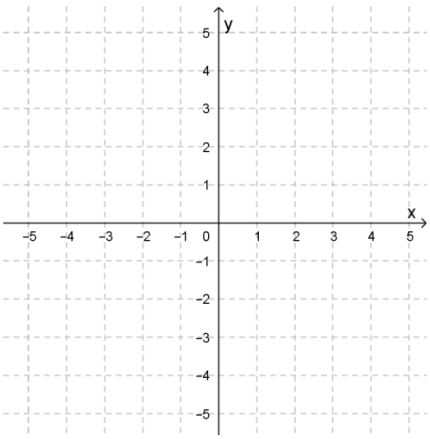
ex: Graph $y = \sqrt{9 - x^2}$. Find the area between the curve and the x-axis.



8. Graph $y = 2 + \sqrt{9 - x^2}$. Find the area between the curve and the x-axis.



9. Graph $y = |x + 1| + 2$ from $x = -4$ to $x = 1$. Find the area between the curve and the x-axis.



Practice at home

10. Find the first 4 terms and the 100th term of $a_n = \frac{(-1)^{n+1}n^3}{3n+1}$.

11. Find the first 4 terms of $a_n = \frac{4n-2}{3^{n-1}}$.

12. Find the first 4 terms of $a_n = \frac{(-1)^n(-2n+5)}{(n+1)^2}$.

13. Evaluate $\sum_{n=1}^{6}(3n+2)$

14. Evaluate $\sum_{i=1}^{5}(-1)^i 2i^2$

15. Evaluate $\sum_{i=1}^{7}(2^i - 3i)$

16. Use the summation formulas to evaluate $\sum_{i=1}^{10}(2i^2 - i)$

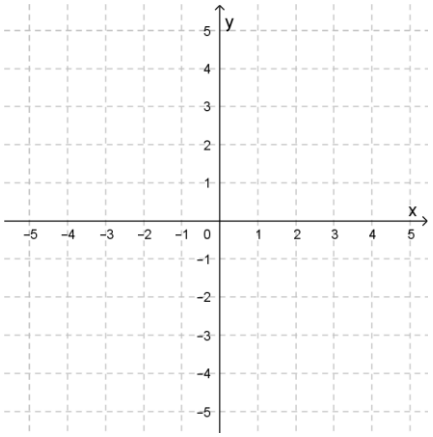
17. Use the summation formulas to evaluate $\sum_{i=1}^{8}(3 - i^3)$

18. Simplify $\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right)$

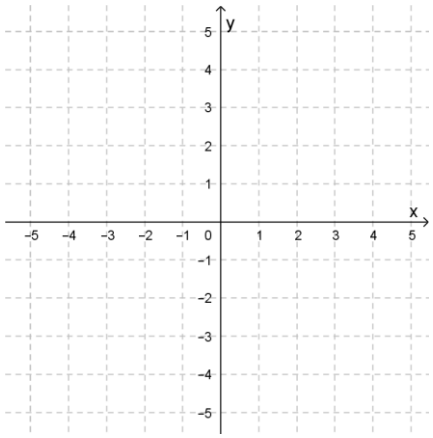
19. Simplify $\sum_{i=1}^{n}\left(\frac{2i}{n}-\left(\frac{2i}{n}\right)^3\right)\cdot\frac{2}{n}$

20. Simplify $\sum_{i=1}^{n}\left(\left(\frac{3i}{n}\right)^2+2\left(\frac{3i}{n}\right)\right)\cdot\frac{3}{n}$

21. Graph $y = 3 + \sqrt{4 - x^2}$. Find the area between the curve and the x-axis.



22. Graph $y = |x + 2| + 3$ from $x = -4$ to $x = 1$. Find the area between the curve and the x-axis.



23. Graph $y = -|x - 3| + 2$ from $x = 1$ to $x = 5$. Find the area between the curve and the x-axis.