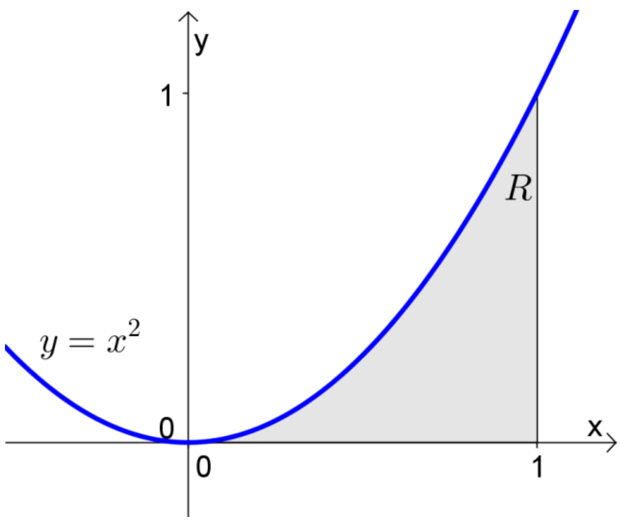
The Area Problem

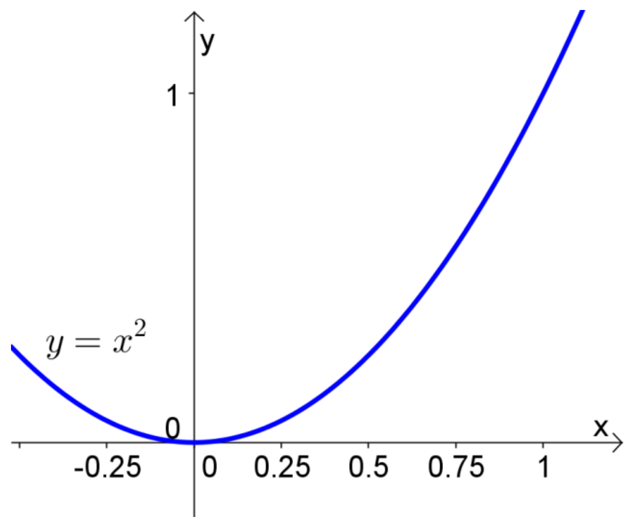
(covers Stewart 5.1)

We know how to calculate areas of some shapes.

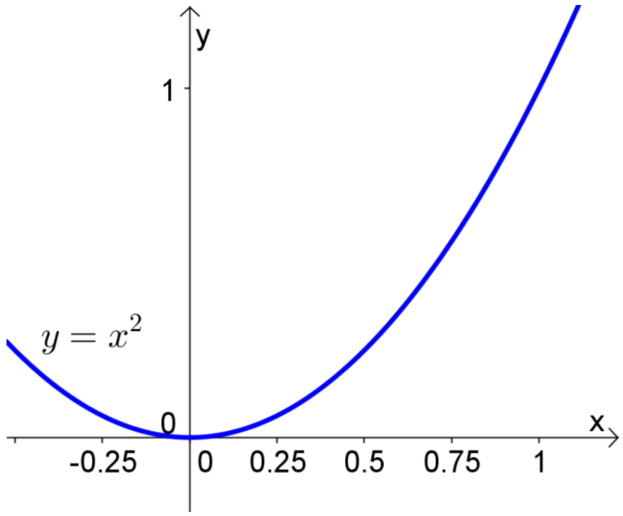
Can we find a more general way to figure out areas? Let's try looking at areas under curves…

We'll start by looking at the area under the curve given by $f(x) = x^2$ from $x = 0$ to $x = 1$. Since we don't have a formula for this area, we'll use rectangles to estimate the area.



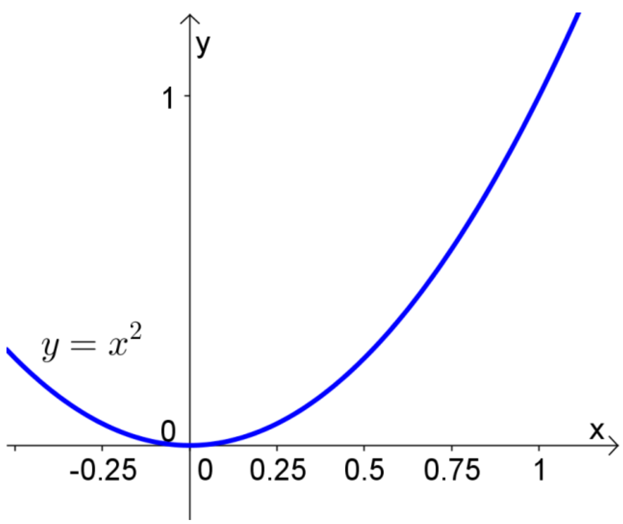


When the heights of the rectangles are determined by the _______________ function value on each subinterval, our area estimation is an _________________, and it always _____________________ the true area under the curve.



When the heights of the rectangles are determined by the _______________ function value on each subinterval, our area estimation is a _________________, and it always _____________________ the true area under the curve.

We could also use the midpoint of each subinterval to get the heights of the rectangles:



Distance Traveled

Suppose we want to find the distance traveled during a certain time period if we know velocity.
If velocity is constant, we have $distance = velocity \times time$.
For example, if we travel at 40 mph for 2 hours, then $distance$ = ______________________.

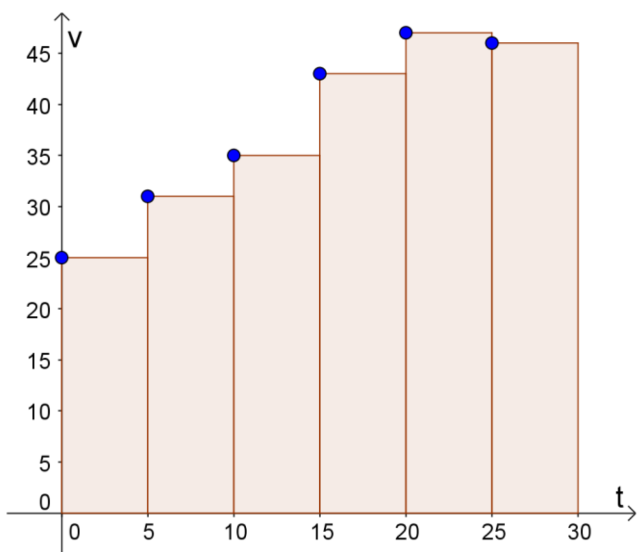
If velocity is not constant, then there's no easy formula. But we can take sample readings of velocity at a bunch of time intervals, and make estimations of the distances traveled on each time interval based on our sample velocities.

Ex 1.

Estimate the distance traveled during the following 30-second interval given the sample velocities.

Time (s)	0	5	10	15	20	25	30
Velocity (ft/s)	25	31	35	43	47	46	41

We can visualize our calculations on a coordinate system:



This gives us our first clue that the area under the curve of a function is somehow related to the antiderivative of the function…