

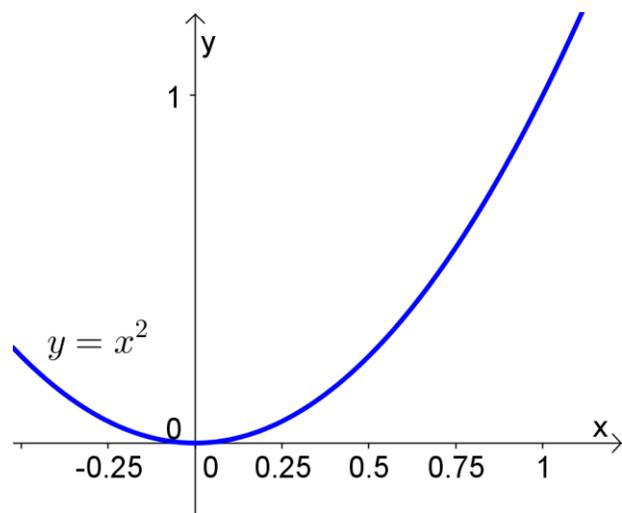
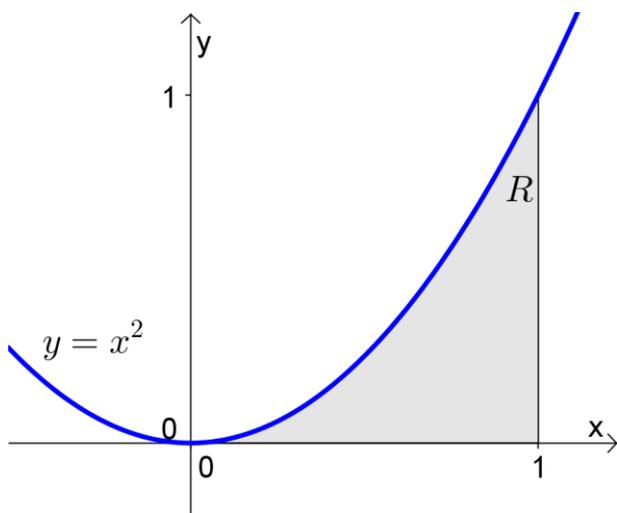
The Area Problem

(covers Stewart 5.1)

We know how to calculate areas of some shapes.

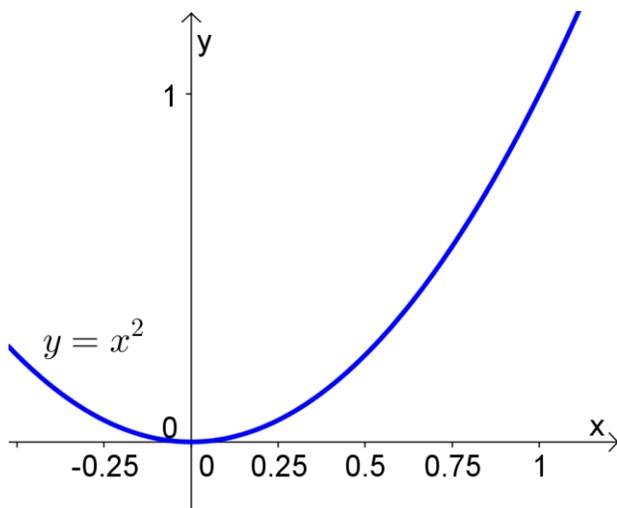
Can we find a more general way to figure out areas? Let's try looking at areas under curves...

We'll start by looking at the area under the curve given by $f(x) = x^2$ from $x = 0$ to $x = 1$. Since we don't have a formula for this area, we'll use rectangles to estimate the area. For the heights of each rectangle, we'll use the maximum function value on each subinterval.



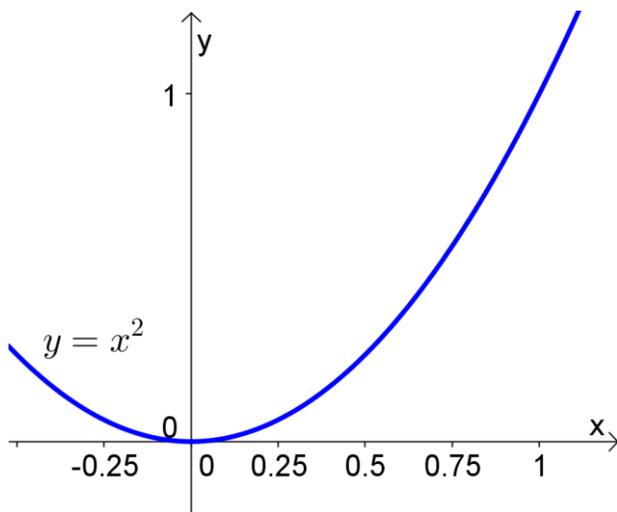
When the heights of the rectangles are determined by the _____ function value on each subinterval, our area estimation is an _____, and it always _____ the true area under the curve.

This time, let's use the minimum function value on each subinterval for the rectangle heights.



When the heights of the rectangles are determined by the _____ function value on each subinterval, our area estimation is a _____, and it always _____ the true area under the curve.

We could also use the midpoint of each subinterval to get the heights of the rectangles:



Distance Traveled

Suppose we want to find the distance traveled during a certain time period if we know velocity.

If velocity is constant, we have $distance = velocity \times time$.

For example, if we travel at 40 mph for 2 hours, then $distance =$ _____.

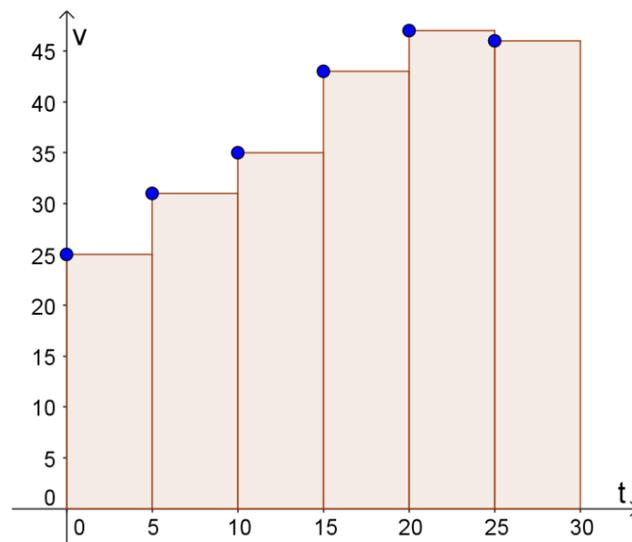
If velocity is not constant, then there's no easy formula. But we can take sample readings of velocity at a bunch of time intervals, and make estimations of the distances traveled on each time interval based on our sample velocities.

Ex 1.

Estimate the distance traveled during the following 30-second interval given the sample velocities.

Time (s)	0	5	10	15	20	25	30
Velocity (ft/s)	25	31	35	43	47	46	41

We can visualize our calculations on a coordinate system:



This gives us our first clue that the area under the curve of a function is somehow related to the antiderivative of the function...