# Integration by Parts

(covers Stewart 7.1)

Let's derive what's called the Integration by Parts formula:

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule for derivatives)

$f(x)g(x) = \int [f(x)g'(x) + g(x)f'(x)]\, dx$

$\int [f(x)g'(x) + g(x)f'(x)]\, dx = f(x)g(x)$

$\int f(x)g'(x)\, dx + \int g(x)f'(x)\, dx = f(x)g(x)$

$\int f(x)g'(x)\, dx = f(x)g(x) - \int g(x)f'(x)\, dx$

Rewrite using $u = f(x)$ and $v = g(x)$, so that $du = f'(x)\, dx$ and $dv = g'(x)\, dx$ to get the **Integration by Parts formula**:

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Let's see how to use it…

**Ex 1.**

$\int \underbrace{x}_{u} \underbrace{\sin x\, dx}_{dv}$

$u = x \qquad dv = \sin x\, dx$

$du = dx \qquad v = -\cos x$

$= \underbrace{(x)}_{u}\underbrace{(-\cos x)}_{v} - \int \underbrace{(-\cos x)}_{v}\underbrace{dx}_{du}$

$= -x\cos x + \int \cos x\, dx$

$= \boxed{-x\cos x + \sin x + C}$

Check:

$\frac{d}{dx}(-x\cos x + \sin x) = -x\cdot(-\sin x) + (-1)\cos x + \cos x$

$= x\sin x$

Note that if we had chosen $u$ and $dv$ differently, it wouldn't work:

$\int \underbrace{(\sin x)}_{u} \cdot \underbrace{x\, dx}_{dv}$

$u = \sin x \qquad dv = x\, dx$

$du = \cos x\, dx \qquad v = \frac{x^2}{2}$

$= (\sin x)\frac{x^2}{2} - \underbrace{\int \frac{x^2}{2}\cos x\, dx}_{\text{Harder to do}}$

The goal is to turn an integral $\int u\,dv$ that you *can't* do into an integral $\int v\,du$ that you *can* do.

**Ex 2.**

$\int \ln x \; dx$

**Ex 3.**

$\int x^2 e^x \, dx$

Here's a quick way to do integration by parts multiple times, called **tabular integration**:

**Ex 4.**

$\int x^2 e^x \, dx$

**Ex 5.**
$\int x^3 \sin 2x \, dx$

Sometimes when doing integration by parts, you'll get back to the original integral.

**Ex 6.**
$\int e^x \cos x \, dx$