Substitution

(covers Stewart 5.5)

The idea for the **substitution method** is to make a substitution so that the integral is simpler.

Ex 1.

Evaluate:

$$\int (x^3 + x)^5 (3x^2 + 1)\, dx$$

Let $u = x^3 + x$

$\frac{du}{dx} = 3x^2 + 1$

$du = (3x^2 + 1)\, dx$

$= \int u^5\, du$

$= \frac{u^6}{6} + C$

$= \frac{(x^3+x)^6}{6} + C$

$\frac{d}{dx}\left(\frac{(x^3+x)^6}{6} + C\right) = (x^3 + x)^5 \cdot \frac{d}{dx}(x^3 + x)$

$= (x^3+x)^5 (3x^2+1)$

The Substitution Rule

If $u = g(x)$ is a differentiable function whose range is an interval I, and f is continuous on I, then

$$\int f(g(x))g'(x)\, dx = \int f(u)\, du$$

$F'(g(x))\, g'(x)$

(**Proof:** Suppose $F' = f$ and $u = g(x)$. Then,

$\int f(g(x))g'(x)\, dx = \int \frac{d}{dx} F(g(x))\, dx = F(g(x)) + C = F(u) + C = \int F'(u)\, du = \int f(u)\, du$.)

$\to F'(g(x)) \cdot g'(x)$

Notes:

To use the substitution method, what you're looking for is a function and its derivative.

The function for u will usually be inside another function.

You want the substitution to make the new integral easier to integrate.

Ex 2.

Evaluate:

$$\int \sec^2(5x + 1) \cdot 5\, dx$$

$u = 5x + 1$

$\frac{du}{dx} = 5$

$du = 5\, dx$

$= \int \sec^2 u\, du$

$= \tan u + C$

$= \tan(5x+1) + C$

Ex 3.

Evaluate:

$$\int \sqrt{2x+1}\,dx$$

Ex 4.

Evaluate:

$$\int x^2 e^{x^3}\,dx$$

Sometimes you need to do some algebra before you can do a substitution.

Ex 5.

Evaluate:

$$\int \frac{dx}{e^x + e^{-x}}$$

Ex 6.

Evaluate:

$$\int \sec x \, dx$$

Ex 7.

Evaluate:

$$\int x\sqrt{2x+1}\, dx$$