

Due date: _____

Name: _____

1. Evaluate the following integrals.

a) $\int \cos x \sqrt{1 + \sin x} dx$

$u = 1 + \sin x$
 $du = \cos x dx$

$= \int \sqrt{u} \cdot du$

$= \int u^{1/2} du$

$= \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3} (1 + \sin x)^{3/2} + C$

(Note: A cloud-shaped note contains the formula $\frac{u^{3/2}}{(3/2)} = \frac{2}{3} u^{3/2}$)

b) $\int e^{2 \cos x} \cdot \sin x dx$

c) $\int 3x e^{x^2+2} dx$

$u = x^2 + 2$
 $du = 2x dx$
 $\frac{1}{2} du = x dx$
 $\frac{3}{2} du = 3x dx$

$= \int e^u \cdot \frac{3}{2} du$

$= \frac{3}{2} \int e^u du$

$= \frac{3}{2} e^u + C$

$= \frac{3}{2} e^{x^2+2} + C$

$$d) \int \frac{\sec x \tan x}{1 + \sec^2 x} dx$$

$$\begin{aligned} e) \int \sqrt{-2x+3} dx & \quad \begin{array}{l} u = -2x + 3 \\ du = -2dx \\ -\frac{1}{2} du = dx \end{array} \\ &= \int \sqrt{u} \cdot \left(-\frac{1}{2} du\right) \\ &= -\frac{1}{2} \int u^{1/2} du \\ &= -\frac{1}{2} \left(\frac{2}{3} u^{3/2}\right) + C \\ &= \boxed{-\frac{1}{3} (-2x+3)^{3/2} + C} \end{aligned}$$

$$f) \int \frac{2}{3-5x} dx$$

$$g) \int \frac{2}{x(\ln x)^3} dx$$

$$h) \int \frac{\log_2 x}{x} dx$$

$$= \int \log_2 x \cdot \frac{1}{x} dx$$

$$= \int u \cdot (\ln 2) du$$

$$= \ln 2 \int u du$$

$$= (\ln 2) \cdot \frac{u^2}{2} + C$$

$$= \boxed{(\ln 2) \frac{(\log_2 x)^2}{2} + C}$$

$$\begin{aligned} u &= \log_2 x \\ du &= \frac{1}{x \ln 2} dx \\ (\ln 2) du &= \frac{1}{x} dx \end{aligned}$$

$$\left(\text{or } \frac{(\log_2 x)^2 \ln 2}{2} + C \right)$$

$$i) \int \frac{2 \sin x \cos x}{1 + \cos^2 x} dx$$

$$j) \int \frac{dx}{(\tan^{-1} x)^2 (1+x^2)}$$

$$= \int \frac{1}{(\tan^{-1} x)^2} \cdot \frac{1}{1+x^2} dx \quad \begin{array}{l} u = \tan^{-1} x \\ du = \frac{1}{1+x^2} dx \end{array}$$

$$= \int \frac{1}{u^2} \cdot du$$

$$= \int u^{-2} du$$

$$= \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = \boxed{-\frac{1}{\tan^{-1} x} + C}$$

Note: $\tan^{-1} x \neq \frac{1}{\tan x}$!



$$k) \int \tan x dx$$

$$l) \int \cot x dx$$

$$= \int \frac{\cos x}{\sin x} dx \quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array}$$

$$= \int \frac{1}{u} \cdot du$$

$$= \ln|u| + C$$

$$= \boxed{\ln|\sin x| + C}$$

$$m) \int (2x^2 + 1)(2x^3 + 3x + 3)^5 dx$$

$$\begin{aligned}
 n) \int \frac{3x^3+6}{(x^4+8x)^5} dx & \quad \begin{aligned} & u = x^4 + 8x \\ & du = (4x^3 + 8) dx \\ & \frac{1}{4} du = (x^3 + 2) dx \\ & \frac{3}{4} du = (3x^3 + 6) dx \end{aligned} \\
 & = \int \frac{1}{u^5} \cdot \frac{3}{4} du \\
 & = \frac{3}{4} \int u^{-5} du \\
 & = \frac{3}{4} \left(\frac{u^{-4}}{-4} \right) + C \\
 & = \frac{-3}{16} \cdot \frac{1}{u^4} + C = \boxed{\frac{-3}{16(x^4+8x)^4} + C}
 \end{aligned}$$

$$o) \int x\sqrt{1-x^2} dx$$

p) $\int x\sqrt{1-x} dx$

$$= \int \underbrace{(1-u)}_{(1-u)u^{1/2}} \underbrace{\sqrt{u}}_{u^{1/2}} \cdot (-du)$$

$$\begin{aligned} u &= 1-x \rightarrow x = 1-u \\ du &= -dx \\ -du &= dx \end{aligned}$$

$$= - \int (u^{1/2} - u^{3/2}) du$$

$$= - \left(\frac{2}{3} u^{3/2} - \frac{2}{5} u^{5/2} \right) + C$$

$$= \boxed{\frac{2}{5} (1-x)^{5/2} - \frac{2}{3} (1-x)^{3/2} + C}$$

q) $\int \frac{x+1}{x^2+1} dx$

$$= \int \left(\frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$= \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$\begin{aligned} u &= x^2+1 \\ du &= 2x dx \\ \frac{1}{2} du &= x dx \end{aligned}$$

$$= \int \frac{1}{u} \cdot \frac{1}{2} du + \tan^{-1} x + C$$

$$= \frac{1}{2} \ln|u| + \tan^{-1} x + C$$

$$= \boxed{\frac{1}{2} \ln|x^2+1| + \tan^{-1} x + C}$$

Q: Which eight-letter word still remains a word after removing each letter from it? **STARTING**

Optional exercises from the Stewart textbook if you'd like more practice:

5.5 (p.418) #1-47 odd

STARING

STRING

SING

SIN

SIN

IN

I