Antiderivatives and Indefinite Integrals

(covers Stewart 4.9 and parts of 5.4)

Sometimes we know the derivative of a function, and want to find the original function. (ex: finding position from velocity.)

F is an __antiderivative__ of f on an interval I if $F'(x) = f(x)$ for all x in I.

Ex 1.

Find an antiderivative for each of the following functions.

$f(x) = 2x$

$F(x) = x^2$ since $F'(x) = 2x = f(x)$

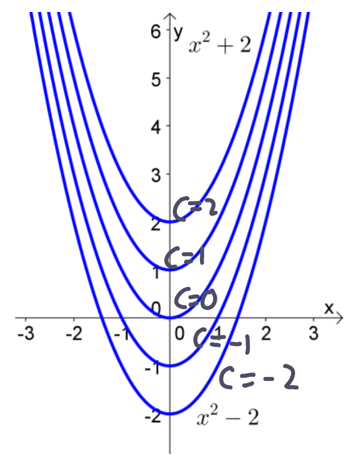
antider. $f \to F$, der. $F \to f$

$g(x) = \cos x$

$G(x) = \sin x$ since $G'(x) = \cos x = g(x)$

$h(x) = \frac{1}{x} + 2e^{2x}$

$H(x) = \ln x + e^{2x}$ since $H'(x) = \frac{1}{x} + 2e^{2x} = h(x)$



Note: The **general antiderivative** of $f(x) = 2x$ is $F(x) = x^2 + \boldsymbol{C}$.

Ex 2.

Find the antiderivative of $f(x) = 3x^2$ that satisfies $F(1) = -1$. $(1, -1)$

$F(x) = x^3 + C$

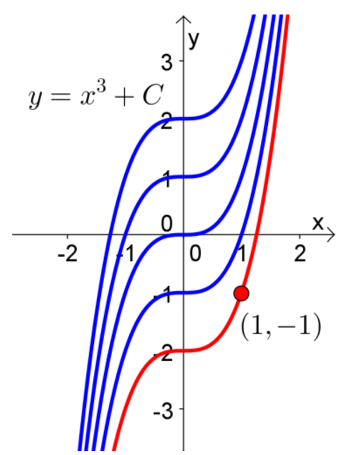
Find C:

$F(1) = -1$

$1^3 + C = -1$

$C = -2$

$F(x) = x^3 - 2$



Let's fill out the following table of antiderivatives:

Function	General antiderivative
x^n	
$\sin x$	
$\cos x$	
$\sec^2 x$	
$\csc^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$
e^x	
$\frac{1}{x}$	
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} x + C$
a^x	$\frac{a^x}{\ln a} + C$

Ex 3.

Find the most general antiderivative for each of the following functions.

$f(x) = \frac{1}{\sqrt{x}}$

$g(x) = \frac{1}{x} + \sin x - 2^x$

$h(x) = \cos 2x + e^{-3x} + 17$

Ex 4.
Find f if $f'(x) = \sec^2 x + \sin x$, and $f(\pi) = 1$.

Note: Since $v'(t) = a(t)$, velocity is an antiderivative of acceleration. And since $s'(t) = v(t)$, position is an antiderivative of velocity.

Ex 5.
A particle is moving with the given data. Find the position of the particle.
$a(t) = 6t + 4,\ v(0) = -6,\ s(0) = 9$

Going forward, we'll use the following notation to represent the general antiderivative of $f(x)$:

$$\int \boldsymbol{f(x)\,dx}$$

This is called the **integral** of f with respect to x.

Ex 6.
Evaluate
$$\int (x^2 - 2x + 5)\,dx$$

Ex 7.
Evaluate
$$\int \left(\frac{5}{x} + \sec^2 3x\right) dx$$

Notes:
$\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$ (That is, the derivative "undoes" the integral.)
$\int \frac{d}{dx}\big(f(x)\big)\,dx = f(x) + C$ (That is, the integral basically "undoes" the derivative.)