Due date: ____________ **Name:** ____________________________________

1. Find the most general antiderivative of each of the following functions.

a) $f(x) = x^{-3}(x^7 - 3x^4 + x + 4)$ (Hint: distribute first)

$f(x) = x^4 - 3x + x^{-2} + 4x^{-3}$

$F(x) = \frac{x^5}{5} - 3\cdot\frac{x^2}{2} + \frac{x^{-1}}{-1} + 4\cdot\frac{x^{-2}}{-2} + C$

$= \boxed{\frac{x^5}{5} - \frac{3x^2}{2} - \frac{1}{x} - \frac{2}{x^2} + C}$

b) $f(x) = 4\sec x \tan x - 3e^x + \frac{5}{x} - \sqrt[3]{x}$

c) $f(x) = 2\csc^2 x + \cos 4x - \frac{2}{x^2} + \frac{5}{\sqrt[3]{x}}$

$f(x) = 2\csc^2 x + \cos 4x - 2x^{-2} + 5x^{-1/3}$

$F(x) = -2\cot x + \frac{1}{4}\sin 4x - 2\cdot\frac{x^{-1}}{-1} + 5\cdot\frac{x^{2/3}}{(2/3)} + C$

$= \boxed{-2\cot x + \frac{1}{4}\sin 4x + \frac{2}{x} + \frac{15x^{2/3}}{2} + C}$

d) $f(x) = \frac{1}{2x} + \frac{2}{\sqrt{x}} - \sin 3x - 4\sec^2 x$

2. Evaluate the following integrals.

a) $\int x^{-3}(x+1)\,dx$

$= \int (x^{-2} + x^{-3})\,dx$

$= \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + C$

$= \boxed{\frac{-1}{x} - \frac{1}{2x^2} + C}$

b) $\int \left(\frac{1}{\sqrt[3]{x}} + \sqrt[4]{x^3}\right) dx$

c) $\int \left(\frac{2}{x} + e^{-5x}\right) dx$

$= \int \left(2 \cdot \frac{1}{x} + e^{-5x}\right) dx$

$= 2 \ln|x| + \frac{e^{-5x}}{-5} + C$

$= \boxed{2 \ln|x| - \frac{1}{5} e^{-5x} + C}$

d) $\int \left(-\frac{\sec^2 4x}{3}\right) dx$

e) $\int \left(2 - \frac{1}{3\sqrt[3]{x}}\right) dx$

$= \int \left(2 - \frac{1}{3} x^{-1/3}\right) dx$

$= 2x - \frac{1}{3} \cdot \frac{x^{2/3}}{(2/3)} + C$

$= \boxed{2x - \frac{1}{2} x^{2/3} + C}$

f) $\int (3^x - \csc 5x \cot 5x)\, dx$

g) $\int (\sin 2x - \csc^2 x + \cosh x + \operatorname{sech}^2 x)\, dx$

$= \boxed{-\frac{1}{2}\cos 2x + \cot x + \sinh x + \tanh x + C}$

h) $\int \left(\frac{2}{x} + \frac{4}{x^2} + \frac{3\sqrt[3]{x}}{4} + x\sqrt{x} - 2\cos 5x - \csc^2 3x\right) dx$

i) $\int \left(x^{10} - \frac{2}{x^3} + \frac{1}{3\sqrt{x}} + \pi - \frac{e^{5x}}{3} - \sec 2x \tan 2x\right) dx$

$= \int \left(x^{10} - 2x^{-3} + \frac{1}{3}x^{-1/2} + \pi - \frac{1}{3}e^{5x} - \sec 2x \tan 2x\right) dx$

$= \frac{x^{11}}{11} - 2\cdot\frac{x^{-2}}{-2} + \frac{1}{3}\cdot\frac{x^{1/2}}{(1/2)} + \pi x - \frac{1}{3}\cdot\frac{e^{5x}}{5} - \frac{1}{2}\sec 2x + C$

$= \boxed{\frac{x^{11}}{11} + \frac{1}{x^2} + \frac{2}{3}\sqrt{x} + \pi x - \frac{e^{5x}}{15} - \frac{1}{2}\sec 2x + C}$

3. Find f if $f''(x) = 3x^2 - x + 5$.

4. Find f if $f'(x) = \frac{3}{\sqrt{1-x^2}}$ and $f(0) = 4$.

$\int f'(x)\,dx = \int 3 \cdot \frac{1}{\sqrt{1-x^2}}\,dx$

$f(x) = 3\sin^{-1}x + C$

$= \boxed{3\sin^{-1}x + 4}$

Find C:

$f(0) = 4$

$3\sin^{-1}0 + C = 4$

$C = 4$

5. For each of the following parts, a particle is moving with the given data. Find a function $s(t)$ that represents the position of the particle as a function of time t.

a) $v(t) = 2t^2 - \sqrt{t},\ s(4) = 5$

$v(t) = 2t^2 - t^{1/2}$

$s(t) = 2 \cdot \frac{t^3}{3} - \frac{t^{3/2}}{(3/2)} + C$

$= \frac{2}{3}t^3 - \frac{2}{3}t^{3/2} + C$

$= \boxed{\frac{2}{3}t^3 - \frac{2}{3}t^{3/2} - \frac{97}{3}}$

Find C:

$s(4) = 5$

$\frac{2}{3}(4)^3 - \frac{2}{3}(4)^{3/2} + C = 5$

$\frac{128}{3} - \frac{16}{3} + C = 5$

$\frac{112}{3} + C = 5 \rightarrow C = \frac{-97}{3}$

b) $v(t) = \frac{4}{1+t^2},\ s(1) = 0$

c) $a(t) = t + \frac{1}{t^2},\ s(2) = 3,\ v(1) = 1\ \ (t > 0)$

$a(t) = t + t^{-2}$

$v(t) = \frac{t^2}{2} + \frac{t^{-1}}{-1} + C$

$= \frac{t^2}{2} - \frac{1}{t} + C$

$= \frac{t^2}{2} - \frac{1}{t} + 1$

Find C:

$v(1) = 1$

$\frac{1^2}{2} - \frac{1}{1} + C = 1$

$C = \frac{3}{2}$

$s(t) = \frac{t^3}{6} - \ln|t| + \frac{3}{2}t + D$

Find D:

$s(2) = 3$

$\frac{2^3}{6} - \ln|2| + 3 + D = 3$

$D = \ln 2 - \frac{4}{3}$

$$\boxed{s(t) = \frac{t^3}{6} - \ln|t| + \frac{3}{2}t + \ln 2 - \frac{4}{3}}$$

d) $a(t) = 5\sin t - 2\cos t,\ s(0) = 0,\ s(2\pi) = 4$

6. True or false: $\int \frac{1}{(3x+5)^2}\,dx = -\frac{1}{3(3x+5)} + C$

$$\frac{d}{dx}\left(-\frac{1}{3(3x+5)}\right) = \frac{d}{dx}\left(-\frac{1}{3}(3x+5)^{-1}\right)$$
$$= -\frac{1}{3} \cdot -(3x+5)^{-2} \cdot 3$$
$$= \frac{1}{(3x+5)^2} \quad \boxed{\text{True}}$$

7. True or false: $\int x \sin x\,dx = -\frac{x^2}{2}\cos x + C$

Q: Solve the following equation with an anagram (that is, rearrange the letters): ELEVEN PLUS TWO = ??

TWELVE PLUS ONE

<u>Optional exercises</u> from the Stewart textbook if you'd like more practice:
4.9 (p.355) #1-21 odd, 25-51 odd, 59-63 odd
5.4 (p.408) #5-15 odd