Optimization Problems

(covers Stewart 4.7)

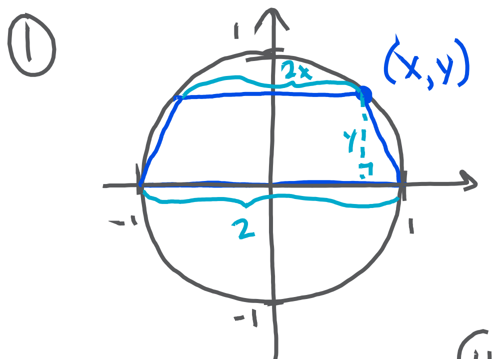
In optimization problems, the goal is to *maximize* or *minimize* something (like volume, area, cost, …).

Optimization problem take practice, but the steps are similar. Here are the basic ones:

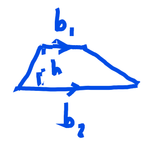
1. Draw a diagram. (Often a coordinate system will help.)
2. Create function that you want to optimize.
3. Find an equation to help rewrite function in one variable.
4. Find absolute max/min for that function using derivatives.

Ex 1.

Find the area of the largest trapezoid that can be inscribed in a circle of radius 1 and whose base is a diameter of the circle. (→ Area function)

① 

② $A = \frac{1}{2}(b_1+b_2)h$



$A(x,y) = \frac{1}{2}(2x+2)y$

$x^2+y^2=1$

$y=\sqrt{1-x^2}$

③ $A(x) = \frac{1}{2}(2x+2)\sqrt{1-x^2}$

$A(x) = (x+1)\cdot\sqrt{1-x^2}$

④ $A'(x) = (x+1)\cdot\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x) + 1\cdot\sqrt{1-x^2}$

$= \frac{-x^2-x}{\sqrt{1-x^2}} + \sqrt{1-x^2}$ $\qquad \sqrt{1-x^2}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} = \frac{1-x^2}{\sqrt{1-x^2}}$

$= \frac{-x^2-x+1-x^2}{\sqrt{1-x^2}}$

$= \frac{-2x^2-x+1}{\sqrt{1-x^2}}$ $\qquad -(2x^2+x-1)$, $\; -(2x-1)(x+1)$

$= \frac{-(2x-1)(x+1)}{\sqrt{1-x^2}}$

$A'=0$:

$-(2x-1)(x+1)=0$

$x=\frac{1}{2}$ $\qquad$ ~~$x=-1$~~ Nonsense!

A' DNE:

$1-x^2 \le 0$

$(1+x)(1-x)\le 0$

$x=-1$ $\qquad x=1$

$(1+x)(1-x)$: $-$ 0 $+$ 0 $-$ (sign line at -1, 1)

~~$x\le -1$~~ or ~~$x \ge 1$~~

Nonsense!

$A\left(\frac{1}{2}\right) = \left(\frac{1}{2}+1\right)\sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \frac{3}{2}\sqrt{\frac{3}{4}}$

$= \boxed{\frac{3\sqrt{3}}{4}}$

Ex 2.

You have been asked to design a 1000-cm^3 can shaped like a right circular cylinder. What dimensions will use the least material?

Ex 3.

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Ex 4.

A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.