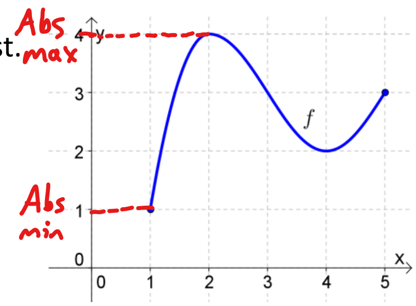
Absolute Extrema and the Extreme Value Theorem

(covers Stewart 4.1)

Calculus can help us find the biggest and smallest values of functions, if they exist.



Suppose we have a function f with domain D.

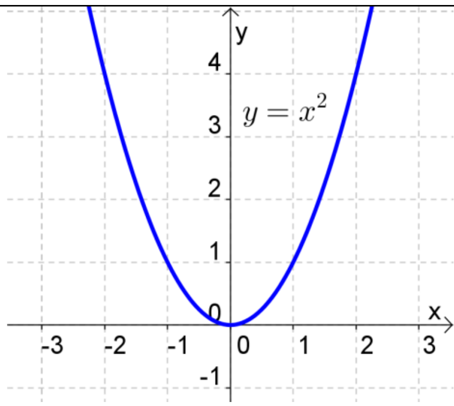
f has an **absolute maximum** value at a point $c \in D$ if $f(c) \geq f(x)$ for all $x \in D$.

f has an **absolute minimum** value at a point $c \in D$ if $f(c) \leq f(x)$ for all $x \in D$.

Absolute maxima and minima are also called global maxima and minima.

Ex 1.

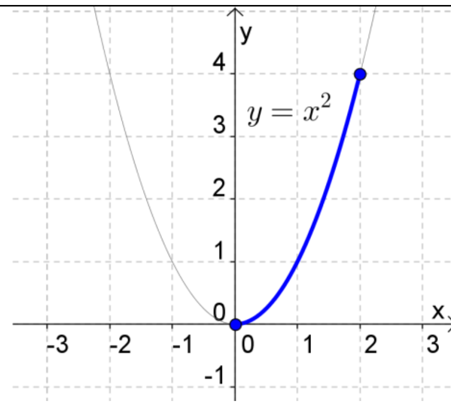
What are the absolute max and absolute min (if any) of $y = x^2$ with the following domains?



Domain: $(-\infty, \infty)$

Abs max: none

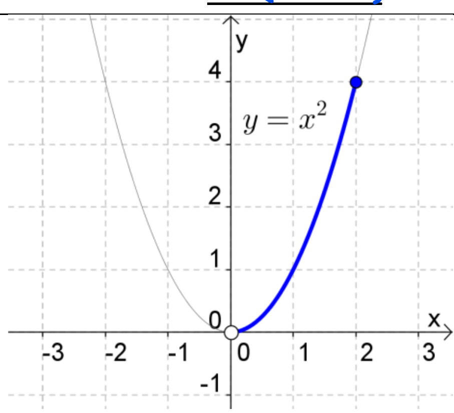
Abs min: 0 (at x=0)



Domain: $[0,2]$

Abs max: 4 (at x=2)

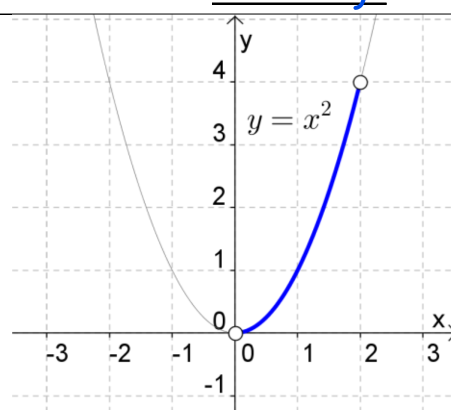
Abs min: 0 (at x=0)



Domain: $(0,2]$

Abs max: 4 (at x=2)

Abs min: none



Domain: $(0,2)$

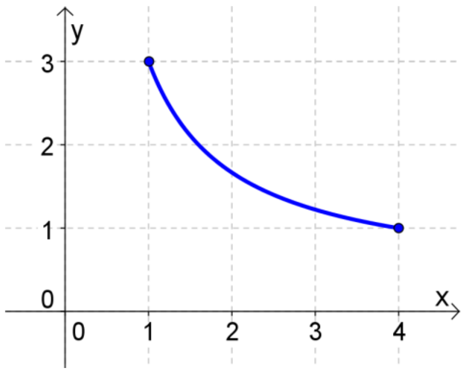
Abs max: none

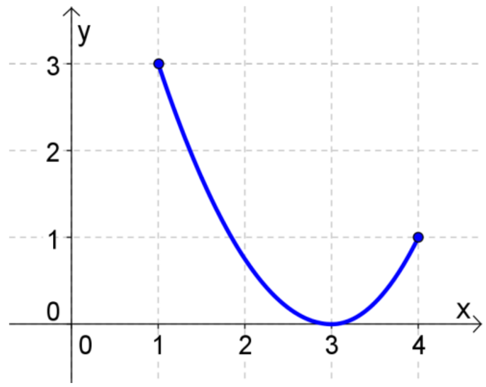
Abs min: none

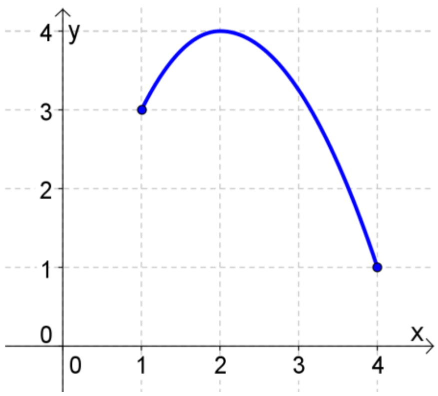
Q: When are we guaranteed to have an absolute max and min?

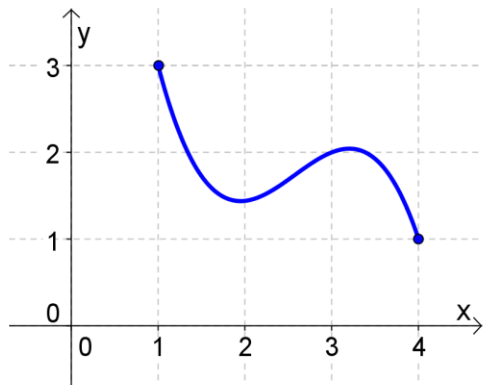
A: The **Extreme Value Theorem** says f must be continuous on a closed interval:

If $f(x)$ is continuous on a closed interval $[a, b]$, then it will have an absolute max and min on $[a, b]$.

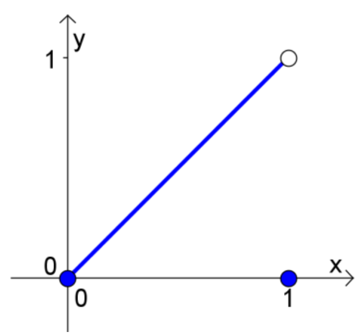






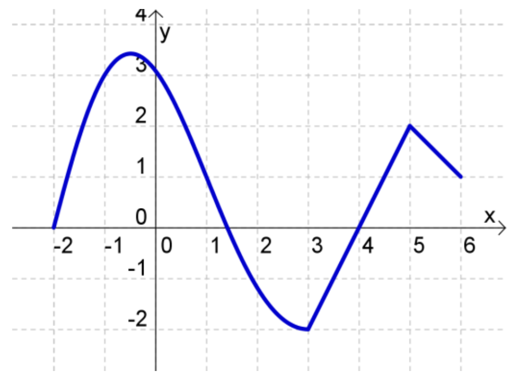


To the right is an example of a function defined on the closed interval $[0,1]$, with no absolute max. Because the function is discontinuous, the Extreme Value Theorem doesn't apply, so we're not guaranteed an absolute max.



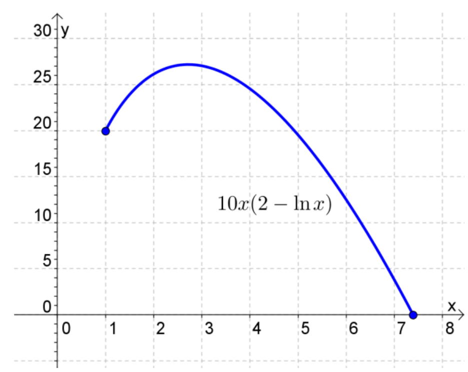
How to find absolute extrema on a closed interval

1. Evaluate f at all critical numbers and endpoints.
2. Take the largest and smallest of these values.



Ex 2.

Find the absolute maximum and minimum values of $f(x) = 10x(2 - \ln x)$ on the interval $[1, e^2]$.



Ex 3.

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2,3]$.