# Graphing using Calculus

(covers Stewart 4.5)

**How to sketch a curve** (using the techniques we know so far):

1. Find **domain**.
2. Find/plot **intercepts**. (if possible)
3. Find/draw **asymptotes**.
4. Find $\boldsymbol{f'}$ and $\boldsymbol{f''}$, and determine when each are **0** or **DNE**.
5. Do a **sign analysis** on $f'$ and $f''$.
6. Find/plot **maxima/minima**, and **inflection points**.
7. Sketch curve.

**Ex 1.**

Sketch the graph of $f(x) = 3x^4 + 4x^3$

(1) Domain: $(-\infty, \infty)$

(2) x-int (set y = 0): $3x^4 + 4x^3 = 0 \rightarrow x^3(3x+4) = 0$ $(0,0)$ $(-\frac{4}{3}, 0)$

y-int (set x = 0): $f(0) = 0$ $(0,0)$

(3) VA: None since f is polynomial.

HA: None " " " "

(4) $f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$

$f' = 0$: $12x^2(x+1) = 0 \rightarrow x = 0, x = -1$

$f'$ DNE: None

$f''(x) = 36x^2 + 24x = 12x(3x+2)$

$f'' = 0$: $12x(3x+2) = 0 \rightarrow x = 0, x = \frac{-2}{3}$

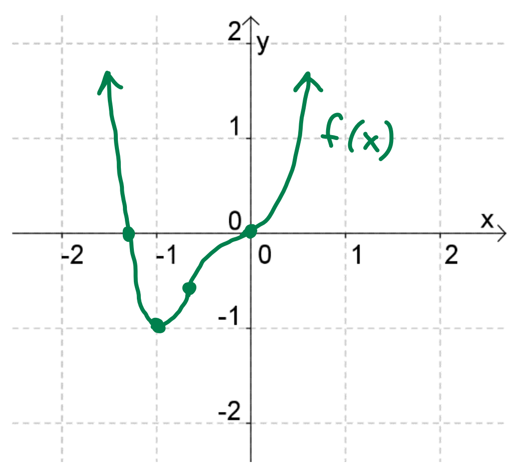
$f''$ DNE: None

(5)

| x | | -1 | | $-\frac{2}{3}$ | | 0 | |
|---|---|---|---|---|---|---|---|
| $f'$ | − | 0 | + | + | + | 0 | + |
| $f''$ | + | + | + | 0 | − | 0 | + |

(6)

| Min | I.P. | I.P. |
|---|---|---|
| $(-1, -1)$ | $(-\frac{2}{3}, \frac{-16}{27})$ | $(0,0)$ |
| ↑ $f(-1)$ | ↑ $f(-\frac{2}{3})$ | ↑ $f(0)$ |



**Review of asymptotes**

1. A **vertical asymptote** $x = a$ happens when $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.

For rational functions $\frac{P(x)}{Q(x)}$, they occur when $Q(x) = 0$ and $P(x) \neq 0$.

ex: Find the vertical asymptotes of $f(x) = \frac{x^3}{x^4 - 16}$.

ex: Does $f(x) = x \ln x$ have a vertical asymptote at $x = 0$?

2. A **horizontal asymptote** $y = L$ happens when $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

For rational functions $\frac{P(x)}{Q(x)}$ …

…if degree($P$) < degree($Q$), then the horizontal asymptote is $y = 0$.

…if degree($P$) = degree($Q$), then the horizontal asymptote is $y = \frac{\text{leading coefficient}(P)}{\text{leading coefficient}(Q)}$.

…if degree($P$) > degree($Q$), then no horizontal asymptotes.

ex: Find the horizontal asymptote of $f(x) = \frac{5x^2}{3x^2 + 7}$.

ex: Find the horizontal asymptote of $f(x) = x^3 e^{-x}$.

**Ex 2.**

Sketch the graph of $f(x) = \frac{x}{(x+1)^2}$