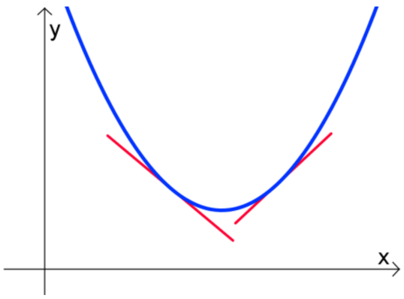
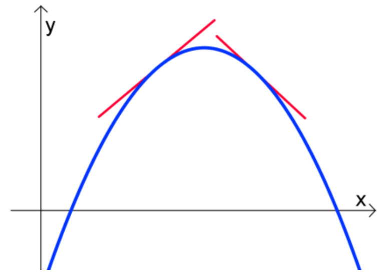
Second Derivatives and Graphs

(covers parts of Stewart 4.3)

The 2nd derivative tells us how the 1st derivative is changing, which tells us how the graph "bends".



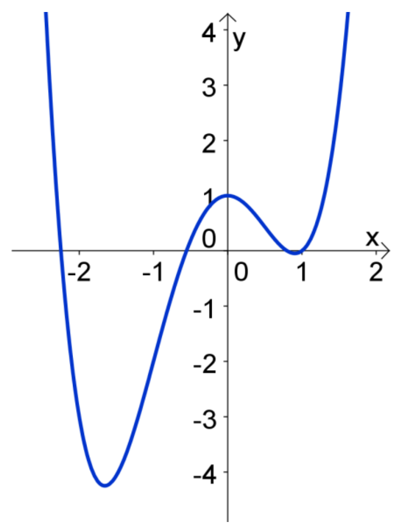


If $\boldsymbol{f''}$ **is positive**, then f' is increasing, and the graph of f is **concave up**.
If $\boldsymbol{f''}$ **is negative**, then f' is decreasing, and the graph of f is **concave down**.

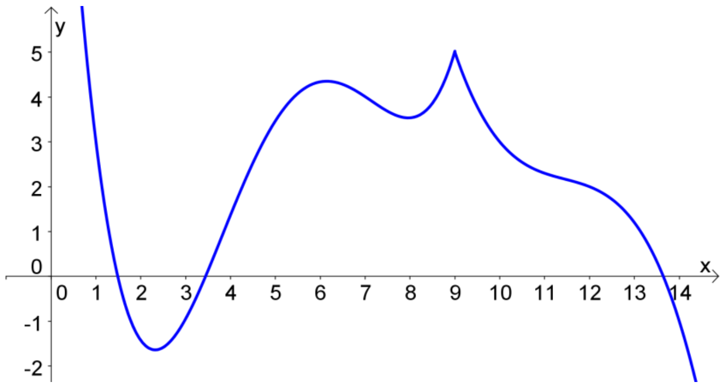
Where might f change from concave up to concave down, or concave down to concave up?

1. When $f''(x) = 0$
2. When $f''(x)$ DNE

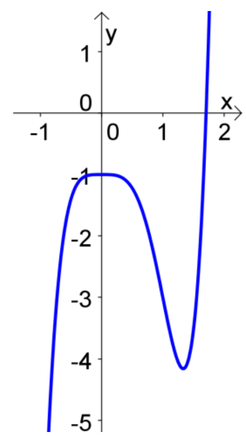
Ex 1.
Find the intervals of concavity of $f(x) = x^4 + x^3 - 3x^2 + 1$.



A point $(c, f(c))$ where the **concavity changes** is called an **inflection point**.
(Again, this can happen if either $f''(c) = 0$ or $f''(c)$ DNE.)



Ex 2.
Find the inflection point(s) of $f(x) = 3x^5 - 5x^4 - 1$.



Second Derivative Test

Suppose f'' is continuous on an open interval about $x = c$, and $f'(c) = 0$.

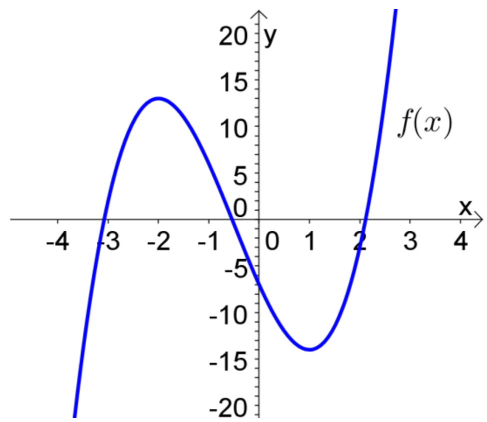
If $\boldsymbol{f'(c) = 0}$ **and** $\boldsymbol{f''(c) > 0}$, then there is a **local minimum** at $x = c$.

If $\boldsymbol{f'(c) = 0}$ **and** $\boldsymbol{f''(c) < 0}$, then there is a **local maximum** at $x = c$.

Note: If $f''(c) = 0$ or $f''(c)$ DNE, then test doesn't say anything (maybe try First Derivative Test).

Ex 3.

Find the local maximum and local minimum values of $f(x) = 2x^3 + 3x^2 - 12x - 7$ using the Second Derivative Test.



Note:

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

Note that here, $f'(x) = 0$ when $x = 0$, but $f''(0) = 0$, so the Second Derivative Test is inconclusive.

Summary:

First Derivative Test – Uses sign of $\boldsymbol{f'}$ **across** a critical number to find local max/min.

Second Derivative Test – Uses sign of $\boldsymbol{f''}$ **at** a critical number to find local max/min.