Due date: _____________ **Name:** ______________________________________

1. Find the intervals of concavity and inflection point(s) of $f(x) = x^4 + x^3 - 3x^2 + 1$.

$f'(x) = 4x^3 + 3x^2 - 6x$

$f''(x) = 12x^2 + 6x - 6$

$= 6(2x^2 + x - 1)$

$= 6(2x-1)(x+1)$

$\underline{f''=0:}$

$6(2x-1)(x+1) = 0$

$x = \frac{1}{2}$ $\quad$ $x = -1$

$\underline{f'' \text{ DNE}:}$

None

-1 $\quad$ $\frac{1}{2}$ $\quad$ x

f'' $\quad + \quad 0 \quad - \quad 0 \quad +$

$\boxed{\begin{array}{l} \text{Concave up: } (-\infty, -1),\ (\frac{1}{2}, \infty) \\ \text{Concave down: } (-1, \frac{1}{2}) \\ \text{Inflection points: } (-1, -2),\ (\frac{1}{2}, \frac{7}{16}) \end{array}}$

$f(-1)$ $\quad$ $f(\frac{1}{2})$

2. Find the intervals of concavity and inflection point(s) of $f(x) = \frac{1}{4-x^2}$.

3. Find the intervals of concavity and inflection point(s) of $f(x) = x^2 e^x$.

$$f'(x) = x^2 e^x + 2x e^x$$

$$f''(x) = x^2 e^x + 2x e^x + 2x e^x + 2e^x$$
$$= x^2 e^x + 4x e^x + 2e^x$$
$$= e^x (x^2 + 4x + 2)$$

$\underline{f'' = 0:}$

$$e^x(x^2+4x+2) = 0$$
$$x^2+4x+2 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{8}}{2}$$
$$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

$\underline{f'' \text{ DNE:}}$

None

	(-3.41) $-2-\sqrt{2}$		(-0.59) $-2+\sqrt{2}$		x
f'' $+$	0	$-$	0	$+$	

$$\boxed{\begin{array}{l} \text{Concave up: } (-\infty, -2-\sqrt{2}),\ (-2+\sqrt{2}, \infty) \\ \text{Concave down } (-2-\sqrt{2}, -2+\sqrt{2}) \\ \text{Inflection points: } (-3.41, 0.38) \\ \quad\quad \text{and } (-0.59, 0.19) \end{array}}$$

4. Find the intervals of concavity and inflection point(s) of $f(x) = \sqrt[3]{x^2+1}$.

5. Find the intervals of concavity and inflection point(s) of $f(x) = \frac{\ln x}{x}$. ← Note: Domain is $(0, \infty)$

$$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$$f''(x) = \frac{x^2 \cdot \left(-\frac{1}{x}\right) - (1 - \ln x) \cdot 2x}{(x^2)^2}$$

$$= \frac{-x - 2x + 2x\ln x}{x^4}$$

$$= \frac{2x\ln x - 3x}{x^4}$$

$$= \frac{2\ln x - 3}{x^3}$$

$f'' = 0$:

$2\ln x - 3 = 0$

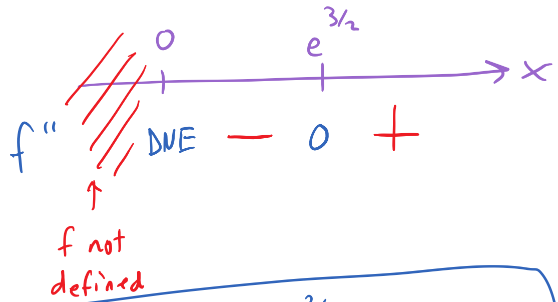
$\ln x = 3/2$

$x = e^{3/2}$

f'' DNE:

$x^3 = 0$ or $x \leq 0$ ← Because of $\ln x$ in f''

$x = 0$



$$\boxed{\begin{array}{l} \text{Concave up: } (e^{3/2}, \infty) \\ \text{Concave down: } (0, e^{3/2}) \\ \text{Inflection point: } \left(e^{3/2}, \frac{3}{2e^{3/2}}\right) \end{array}}$$

6. Find the intervals of concavity and inflection point(s) of $f(x) = \cos x - \sin x$ in the interval $[0, 2\pi]$.

7. Find the intervals of concavity and inflection point(s) of $f(x) = 6x^{1/3} + 3x^{4/3}$.

$f'(x) = 2x^{-2/3} + 4x^{1/3}$

$f''(x) = -\frac{4}{3}x^{-5/3} + \frac{4}{3}x^{-2/3}$

$= -\frac{4}{3}x^{-5/3}(1-x)$

$= \frac{4}{3}x^{-5/3}(x-1)$

$= \frac{4(x-1)}{3x^{5/3}}$

$f''=0$:

$4(x-1)=0$

$x=1$

f'' DNE:

$3x^{5/3}=0$

$x=0$

x: 0, 1

f'': + DNE − 0 +

$$\boxed{\begin{array}{l}\text{Concave up: } (-\infty,0),\ (1,\infty) \\ \text{Concave down: } (0,1) \\ \text{Inflection points: } (0,0) \text{ and } (1,9)\end{array}}$$

8. Find all points where $f(x) = 2x^3 + 3x^2 - 12x - 7$ has a local maximum and local minimum using the Second Derivative Test.

9. Find all points where $f(x) = 2x + 1 + \frac{2}{x}$ has a local maximum and local minimum using the Second Derivative Test.

$f'(x) = 2 - \frac{2}{x^2}$

$= \frac{2x^2 - 2}{x^2}$

$f' = 0$:

$2x^2 - 2 = 0$

$x^2 = 1$

$x = 1, x = -1$

$f''(x) = \frac{4}{x^3}$

Second Derivative Test:

$f''(-1) = \frac{4}{(-1)^3} = -4$

Negative, so $\boxed{\text{local max at } (-1, -3)}$

$f(-1)$

$f''(1) = \frac{4}{(1)^3} = 4$

Positive, so $\boxed{\text{local min at } (1, 5)}$

$f(1)$

10. Find all points where $f(x) = x^3 \ln x$ has a local maximum and local minimum using the Second Derivative Test.

Q: Why couldn't the angle get a loan? It couldn't get its parents to cosign!
(cosine)

Optional exercises from the Stewart textbook if you'd like more practice:
4.3 (p.300) #9-17 odd (part c only), 37-47 (part c only)