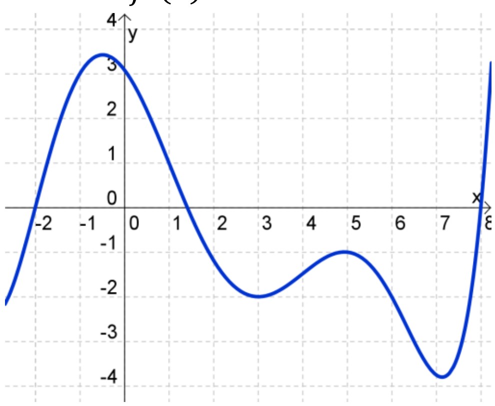
First Derivatives and Graphs

(covers parts of Stewart 4.1 and 4.3)

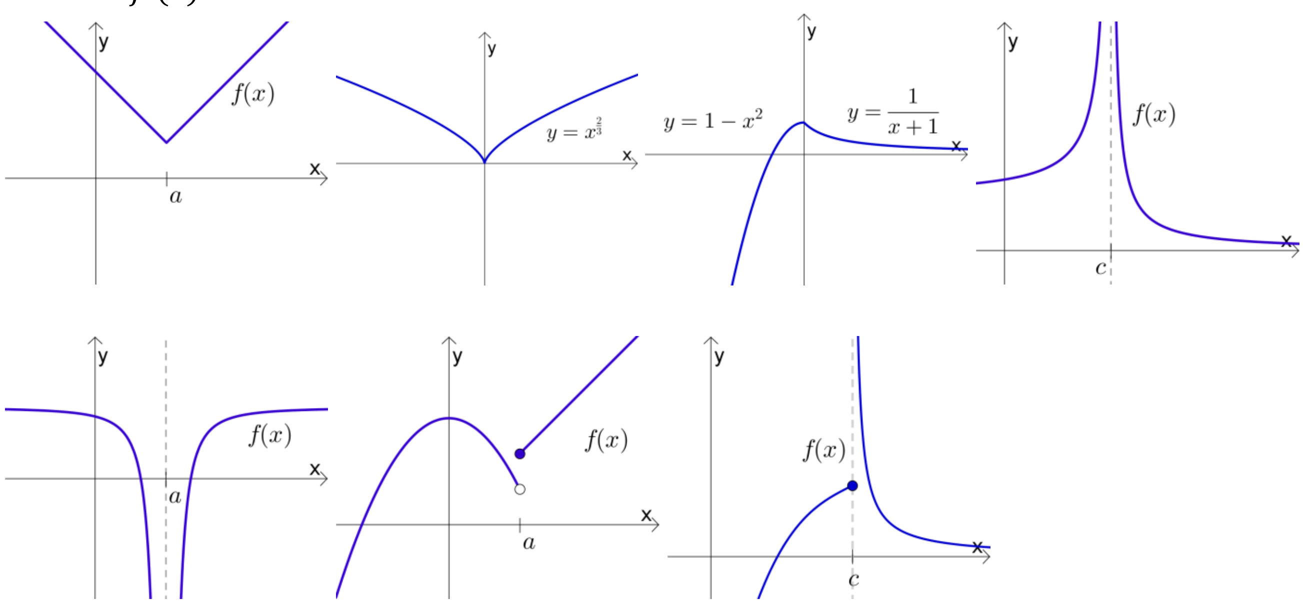
If $\boldsymbol{f'(x) > 0}$ **(positive)** on an interval, then f is **increasing** on that interval.
If $\boldsymbol{f'(x) < 0}$ **(negative)** on an interval, then f is **decreasing** on that interval.

Where might $f(x)$ change from increasing to decreasing, or decreasing to increasing?

1. When $f'(x) = 0$

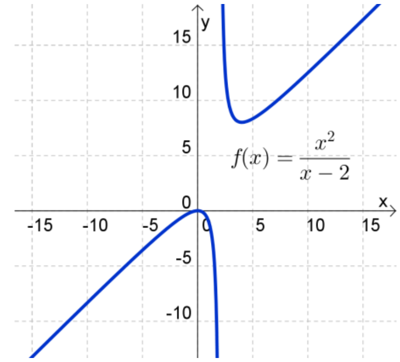


2. When $f'(x)$ DNE



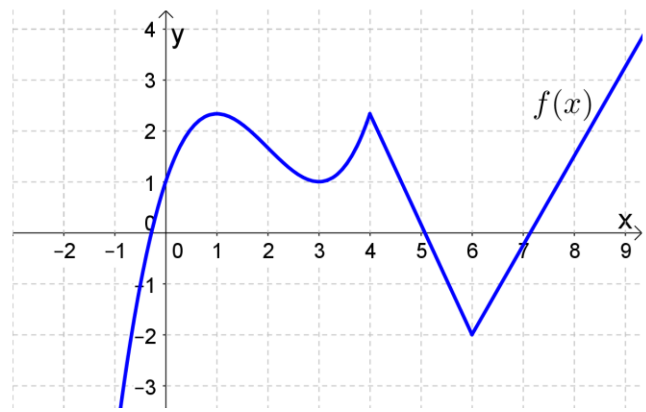
Ex 1.

Find the intervals on which $f(x) = \frac{x^2}{x-2}$ is increasing or decreasing.



Local Extrema

Graphs of functions often have "peaks" and "valleys."
These are called **local extrema**.
A "peak" is called a **local maximum**.
A "valley" is called a **local minimum**.



Here are the more formal definitions for a local max and local min of a function f with domain D:

- f has a **local maximum** at $c \in D$ if $f(c) \geq f(x)$ for all $x \in D$ in some open interval containing c.
- f has a **local minimum** at $c \in D$ if $f(c) \leq f(x)$ for all $x \in D$ in some open interval containing c.

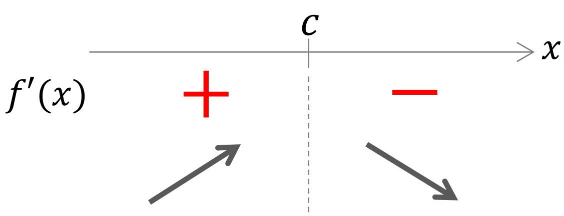
To have a local max, the function needs to be increasing and then decreasing.
To have a local min, the function needs to be decreasing and then increasing.

So, where might $f(x)$ have a local max or a local min?

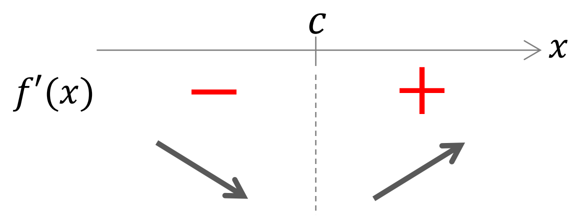
1. When $f'(x) = 0$
2. When $f'(x)$ DNE

First Derivative Test

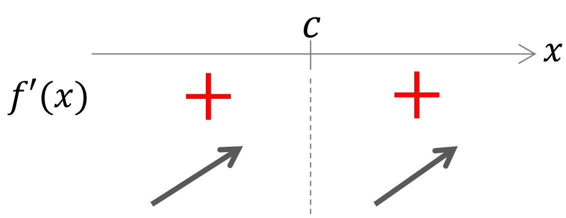
Suppose $f(c)$ is defined, and $f'(c) = 0$ or $f'(c)$ DNE.



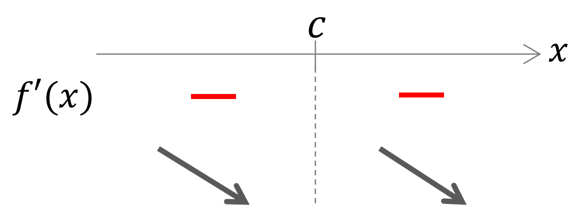
Local
max at
$(c, f(c))$



Local
min at
$(c, f(c))$



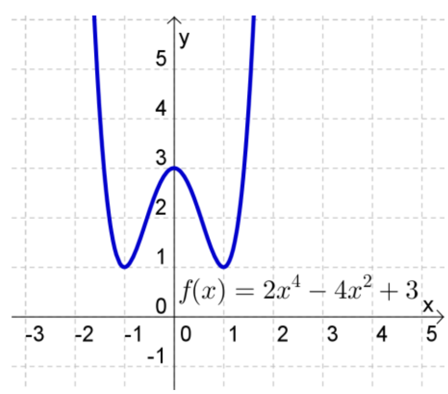
Not a local
extremum



Not a local
extremum

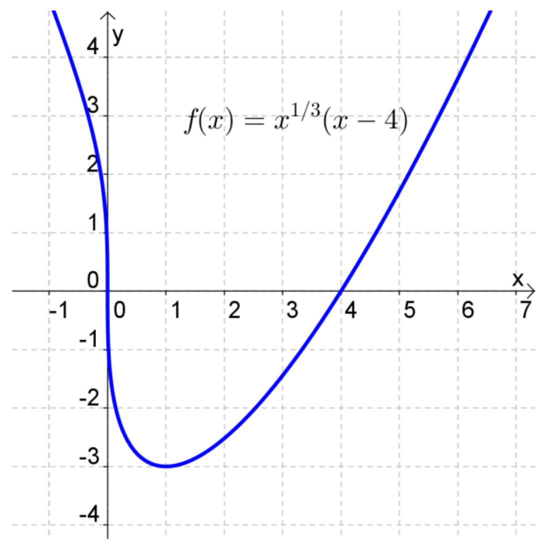
Ex 2.

Find the local maximum and minimum points of $f(x) = 2x^4 - 4x^2 + 3$.



Ex 3.

Find the intervals on which $f(x) = x^{1/3}(x-4)$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.



Note:

f has a **critical number** at c if $f(c)$ is defined, and $f'(c) = 0$ or $f'(c)$ DNE.

The critical numbers of a function give us a list of all possible candidates for local mins/maxs.