

First Derivatives and Graphs

(covers parts of Stewart 4.1 and 4.3)

Recall from Precalculus:

Let $f(x)$ be defined on the interval $a < x < b$. Suppose x_1 and x_2 are in that interval.

f is increasing on that interval if $f(x_2) > f(x_1)$ whenever $x_2 > x_1$ (so, increasing = up and right)

f is decreasing on that interval if $f(x_2) < f(x_1)$ whenever $x_2 > x_1$ (so, decreasing = down and right)

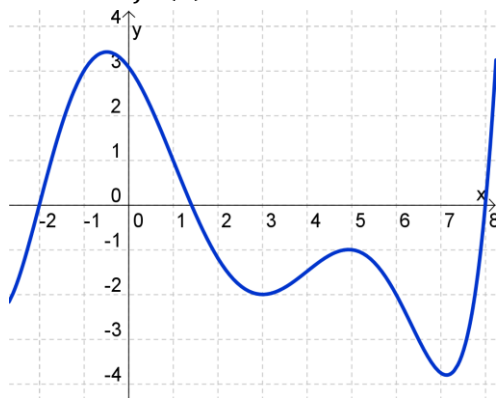
Since $f'(x)$ is the slope of the tangent line at any point on the graph of $f(x)$, $f'(x)$ tells us if $f(x)$ is increasing (positive slope) or decreasing (negative slope). In other words...

If $f'(x) > 0$ (**positive**) on an interval, then f is **increasing** on that interval.

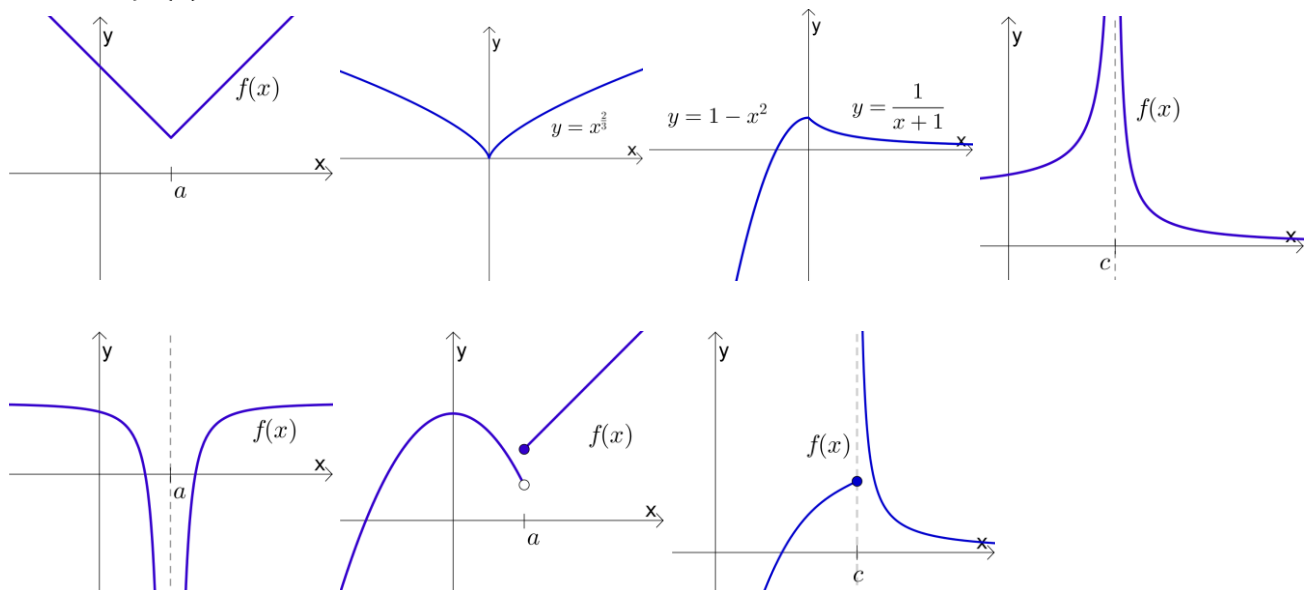
If $f'(x) < 0$ (**negative**) on an interval, then f is **decreasing** on that interval.

Where might $f(x)$ change from increasing to decreasing, or decreasing to increasing?

1. When $f'(x) = 0$

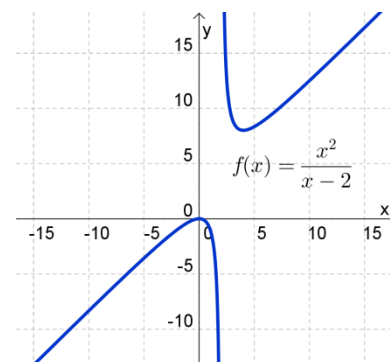


2. When $f'(x)$ DNE



Ex 1.

Find the intervals on which $f(x) = \frac{x^2}{x-2}$ is increasing or decreasing.

**Local Extrema**

Graphs of functions often have “peaks” and “valleys.”

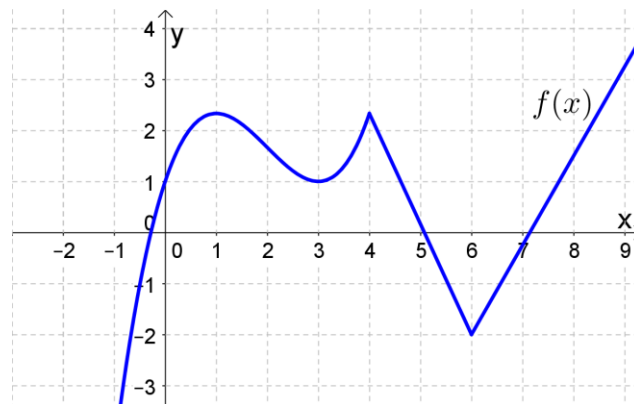
These are called **local extrema**.

A “peak” is called a **local maximum**.

A “valley” is called a **local minimum**.

Here are the more formal definitions for a local max and local min of a function f with domain D :

- f has a **local maximum** at $c \in D$ if $f(c) \geq f(x)$ for all $x \in D$ in some open interval containing c .
- f has a **local minimum** at $c \in D$ if $f(c) \leq f(x)$ for all $x \in D$ in some open interval containing c .



To have a local max, the function needs to be increasing and then decreasing.

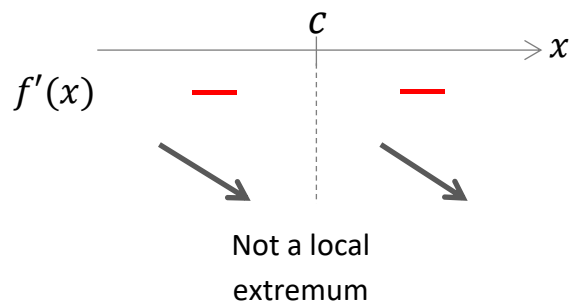
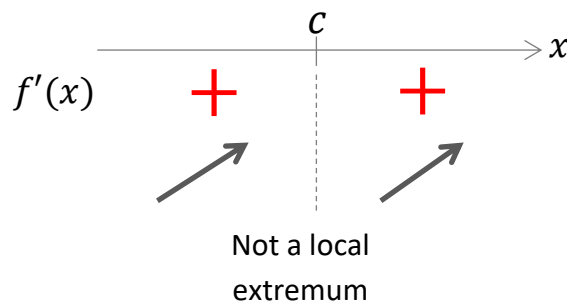
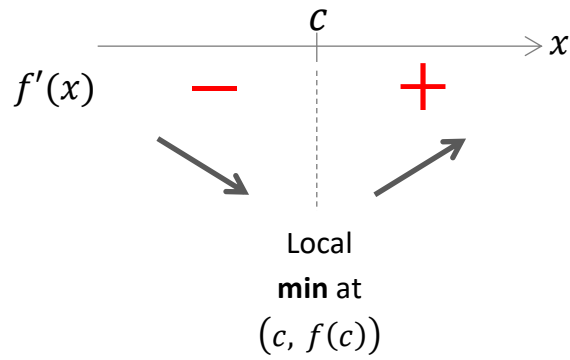
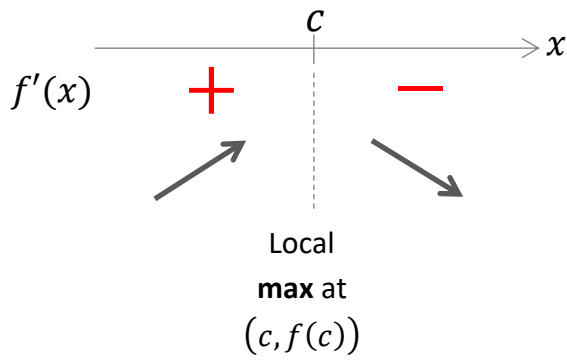
To have a local min, the function needs to be decreasing and then increasing.

So, where might $f(x)$ have a local max or a local min?

1. When $f'(x) = 0$
2. When $f'(x)$ DNE

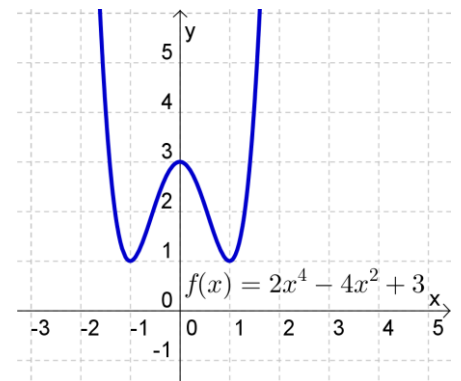
First Derivative Test

Suppose $f(c)$ is defined, and $f'(c) = 0$ or $f'(c)$ DNE.



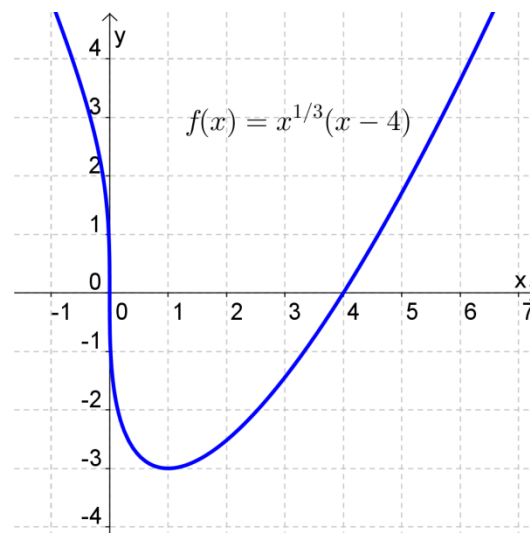
Ex 2.

Find the local maximum and minimum points of $f(x) = 2x^4 - 4x^2 + 3$.



Ex 3.

Find the intervals on which $f(x) = x^{1/3}(x - 4)$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

**Note:**

f has a **critical number** at c if $f(c)$ is defined, and $f'(c) = 0$ or $f'(c)$ DNE.

The critical numbers of a function give us a list of all possible candidates for local mins/maxs.