

Due date: \_\_\_\_\_

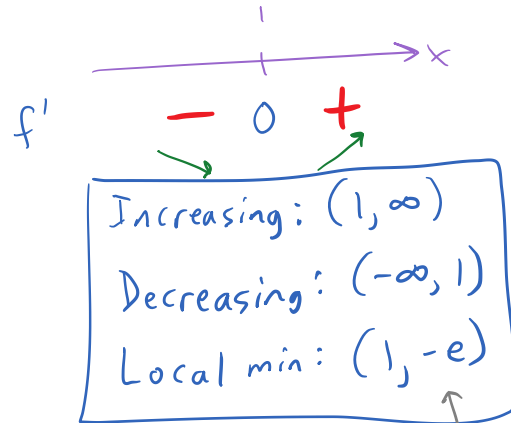
Name: \_\_\_\_\_

1. Find the intervals on which  $f(x) = (x - 2)e^x$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

$$\begin{aligned} f'(x) &= (x-2)e^x + (1)e^x \\ &= xe^x - 2e^x + e^x \\ &= xe^x - e^x \\ &= e^x(x-1) \end{aligned}$$

$$\begin{aligned} \underline{f'(x)=0:} \\ e^x(x-1) = 0 \rightarrow x=1 \end{aligned}$$

$$\begin{aligned} \underline{f'(x) \text{ DNE:}} \\ \text{Nowhere} \end{aligned}$$



$$f(1) = (1-2)e^1 = -e$$

2. Find the intervals on which  $f(x) = x^4 + 4x^3 + 4x^2$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

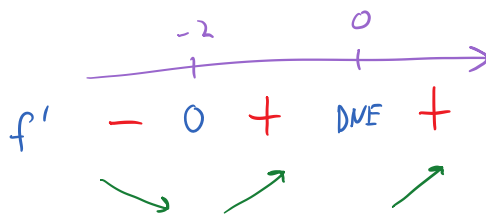
3. Find the intervals on which  $f(x) = x^{1/3}(x + 8)$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

$$f(x) = x^{4/3} + 8x^{1/3}$$

$$\begin{aligned} f'(x) &= \frac{4}{3}x^{1/3} + \frac{8}{3}x^{-2/3} \\ &= \frac{4}{3}x^{-2/3}(x + 2) \\ &= \frac{4(x+2)}{3x^{2/3}} \end{aligned}$$

$$\begin{aligned} \underline{f' = 0:} \\ 4(x+2) = 0 \\ \downarrow \\ x = -2 \end{aligned}$$

$$\begin{aligned} \underline{f' \text{ DNE:}} \\ 3x^{2/3} = 0 \\ x = 0 \end{aligned}$$



Increasing:  $(-2, \infty)$   
 Decreasing:  $(-\infty, -2)$   
 Local min:  $(-2, -6\sqrt[3]{2})$

$$\begin{aligned} f(-2) &= (-2)^{1/3}(-2+8) \\ &= 6 \cdot \sqrt[3]{-2} \\ &= -6\sqrt[3]{2} \end{aligned}$$

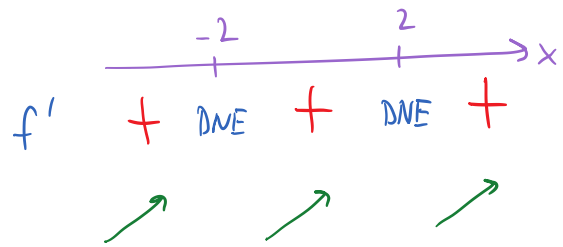
4. Find the intervals on which  $f(x) = x^{4/3} - x^{1/3}$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

5. Find the intervals on which  $f(x) = \frac{x}{4-x^2}$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

$$\begin{aligned} f'(x) &= \frac{(4-x^2)(1) - x(-2x)}{(4-x^2)^2} \\ &= \frac{4-x^2+2x^2}{(4-x^2)^2} \\ &= \frac{x^2+4}{(4-x^2)^2} \end{aligned}$$

$$\begin{aligned} \frac{f'=0:}{x^2+4=0} \\ \text{Nowhere} \end{aligned}$$

$$\begin{aligned} \frac{f' \text{ DNE:}}{(4-x^2)^2=0} \\ 4-x^2=0 \\ (2+x)(2-x)=0 \\ \downarrow \quad \downarrow \\ x=-2 \quad x=2 \end{aligned}$$



Increasing:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Decreasing: Nowhere

No local max/min

6. Find the intervals on which  $f(x) = \ln(x^2 + 1)$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

7. Find the intervals on which  $f(x) = e^{-x^2}$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

$$f'(x) = e^{-x^2} \cdot (-2x)$$

$$= -2xe^{-x^2}$$

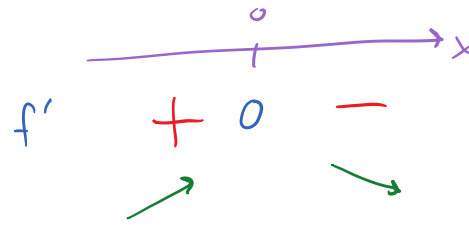
$$\underline{f'=0:}$$

$$-2xe^{-x^2} = 0$$

$$\downarrow$$

$$x=0$$

$f'$  DNE:  
Nowhere



Increasing:  $(-\infty, 0)$

Decreasing:  $(0, \infty)$

Local max:  $(0, 1)$

8. Find the intervals on which  $f(x) = x^3 \ln x$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

9. Find the intervals on which  $f(x) = \tan^{-1}(x^2 - 1)$  is increasing or decreasing. Also, find all points where  $f$  has a local maximum or local minimum.

$$f'(x) = \frac{1}{1+(x^2-1)^2} \cdot 2x$$

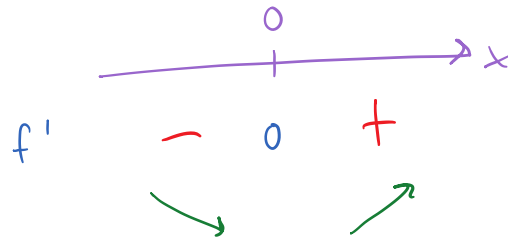
$$= \frac{2x}{1+(x^2-1)^2}$$

$$\frac{f'=0:}{2x=0}$$

$$x=0$$

$$\frac{f' \text{ DNE:}}{1+(x^2-1)^2=0}$$

Nowhere  
(since LHS is positive,  
not zero)



Increasing:  $(0, \infty)$   
 Decreasing:  $(-\infty, 0)$   
 Local min:  $(0, -\frac{\pi}{4})$

Q: What holds water yet is full of holes? *A sponge!*

Optional exercises from the Stewart textbook if you'd like more practice:  
 4.3 (p.300) #9-17 odd (parts a and b), 37-47 (parts a and b)