

Due date: _____

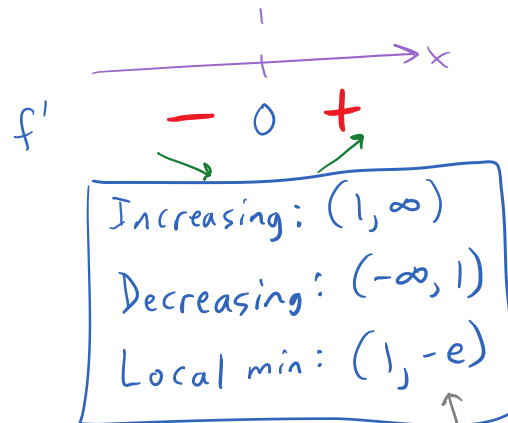
Name: _____

1. Find the intervals on which $f(x) = (x - 2)e^x$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

$$\begin{aligned} f'(x) &= (x-2)e^x + (1)e^x \\ &= xe^x - 2e^x + e^x \\ &= xe^x - e^x \\ &= e^x(x-1) \end{aligned}$$

$$\begin{aligned} \underline{f'(x)=0:} \\ e^x(x-1) = 0 \rightarrow x=1 \end{aligned}$$

$$\begin{aligned} \underline{f'(x) \text{ DNE:}} \\ \text{Nowhere} \end{aligned}$$



$$f(1) = (1-2)e^1 = -e$$

2. Find the intervals on which $f(x) = x^4 + 4x^3 + 4x^2$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

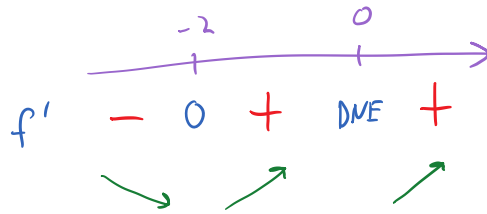
3. Find the intervals on which $f(x) = x^{1/3}(x + 8)$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

$$f(x) = x^{4/3} + 8x^{1/3}$$

$$\begin{aligned} f'(x) &= \frac{4}{3}x^{1/3} + \frac{8}{3}x^{-2/3} \\ &= \frac{4}{3}x^{-2/3}(x+2) \\ &= \frac{4(x+2)}{3x^{2/3}} \end{aligned}$$

$$\begin{aligned} \underline{f' = 0:} \\ 4(x+2) &= 0 \\ \downarrow \\ x &= -2 \end{aligned}$$

$$\begin{aligned} \underline{f' \text{ DNE:}} \\ 3x^{2/3} &= 0 \\ x &= 0 \end{aligned}$$



Increasing: $(-2, \infty)$
 Decreasing: $(-\infty, -2)$
 Local min: $(-2, -6\sqrt[3]{2})$

$$\begin{aligned} f(-2) &= (-2)^{1/3}(-2+8) \\ &= 6 \cdot \sqrt[3]{-2} \\ &= -6\sqrt[3]{2} \end{aligned}$$

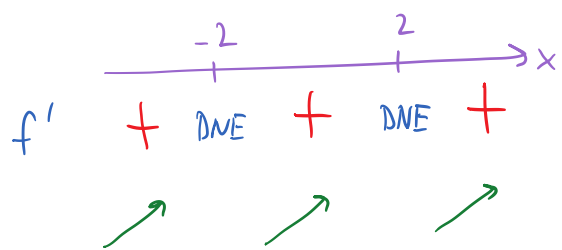
4. Find the intervals on which $f(x) = x^{4/3} - x^{1/3}$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

5. Find the intervals on which $f(x) = \frac{x}{4-x^2}$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

$$\begin{aligned} f'(x) &= \frac{(4-x^2)(1) - x(-2x)}{(4-x^2)^2} \\ &= \frac{4-x^2+2x^2}{(4-x^2)^2} \\ &= \frac{x^2+4}{(4-x^2)^2} \end{aligned}$$

$$\begin{aligned} \frac{f'=0:}{x^2+4=0} \\ \text{None} \end{aligned}$$

$$\begin{aligned} \frac{f' \text{ DNE:}}{(4-x^2)^2=0} \\ 4-x^2=0 \\ (2+x)(2-x)=0 \\ \downarrow \quad \downarrow \\ x=-2 \quad x=2 \end{aligned}$$



Increasing: $(-\infty, -2), (-2, 2), (2, \infty)$

Decreasing: Nowhere

No local max/min

6. Find the intervals on which $f(x) = \ln(x^2 + 1)$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

7. Find the intervals on which $f(x) = e^{-x^2}$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

$$f'(x) = e^{-x^2} \cdot (-2x)$$

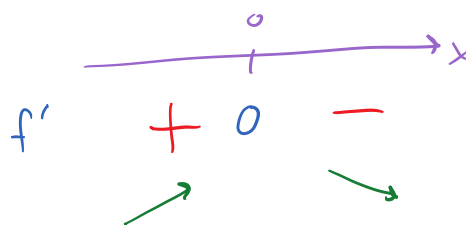
$$= -2xe^{-x^2}$$

$$\frac{f' = 0:}{-2xe^{-x^2} = 0}$$

↓

$$x = 0$$

$$\frac{f' \text{ DNE:}}{\text{None}}$$



Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Local max: $(0, 1)$

8. Find the intervals on which $f(x) = x^3 \ln x$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

9. Find the intervals on which $f(x) = \tan^{-1}(x^2 - 1)$ is increasing or decreasing. Also, find all points where f has a local maximum or local minimum.

$$f'(x) = \frac{1}{1+(x^2-1)^2} \cdot 2x$$

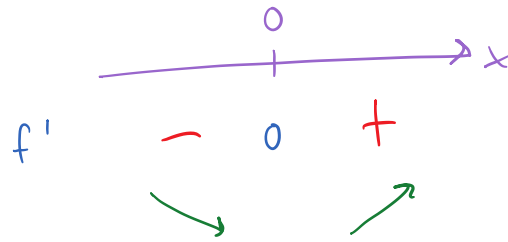
$$= \frac{2x}{1+(x^2-1)^2}$$

$$\frac{f'=0:}{2x=0}$$

$$x=0$$

$$\frac{f' \text{ DNE:}}{1+(x^2-1)^2=0}$$

None
(since LHS is positive,
not zero)



Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

Local min: $(0, -\frac{\pi}{4})$

Q: What holds water yet is full of holes? *A sponge!*

Optional exercises from the Stewart textbook if you'd like more practice:

4.3 (p.300) #9-17 odd (parts a and b), 37-47 (parts a and b)