Due date: ____________ **Name:** ______________________________

1. Use L'Hospital's Rule to find the following limits.

a) $\lim\limits_{x \to 0} \frac{\tan 3x}{x} \leftarrow \frac{0}{0}$

L'Hospital

$= \lim\limits_{x \to 0} \frac{3\sec^2 3x}{1}$

$= \frac{3\sec^2 3(0)}{1}$

$= \boxed{3}$

b) $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

c) $\lim\limits_{x \to 0^+} \frac{\cos x - 1}{x^3} \leftarrow \frac{0}{0}$

L'Hospital

$= \lim\limits_{x \to 0^+} \frac{-\sin x}{3x^2} \leftarrow \frac{0}{0}$

L'Hospital

$= \lim\limits_{x \to 0^+} \frac{-\cos x}{6x}$

$\frac{\text{top} \to -1}{\text{bot} \to 0, \text{pos}}$

$= \boxed{-\infty}$

d) $\lim\limits_{x \to 0^+} \frac{e^x - 1}{x \cos x}$

e) $\lim_{x\to\infty} \frac{e^x}{x^2}$ $\leftarrow \frac{\infty}{\infty}$

L'Hospital

$= \lim_{x\to\infty} \frac{e^x}{2x}$ $\leftarrow \frac{\infty}{\infty}$

L'Hospital

$= \lim_{x\to\infty} \frac{e^x}{2}$

$= \boxed{\infty}$

f) $\lim_{x\to\infty} \frac{x+\ln x}{x\ln x}$

g) $\lim_{x\to 0^+} \sqrt{x}\ln x$ $\leftarrow 0\cdot\infty$

$= \lim_{x\to 0^+} \frac{\ln x}{\left(\frac{1}{x^{1/2}}\right)}$ $\leftarrow \frac{\infty}{\infty}$

L'Hospital

$= \lim_{x\to 0^+} \frac{\left(\frac{1}{x}\right)}{\left(\frac{-1}{2x^{3/2}}\right)}$

$\frac{1}{x}\cdot\frac{2x^{3/2}}{-1} = -2x^{1/2}$

$= \lim_{x\to 0^+} \left(-2x^{1/2}\right)$

$= -2(0)^{1/2}$

$= \boxed{0}$

h) $\lim_{x\to 0}(x \csc x)$

i) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$ $\leftarrow \infty \cdot 0$

$= \lim_{x\to\infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$ L'Hospital

$= \lim_{x\to\infty} \frac{\left(\cos \frac{1}{x}\right) \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(-\frac{1}{x^2}\right)}}$

$= \lim_{x\to\infty} \cos \frac{1}{x}$

$= \boxed{1}$

j) $\lim_{x\to 0^+}\left(\csc x - \frac{1}{x}\right)$

k) $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$ ← $\infty-\infty$

$= \lim_{x\to1^+} \frac{x\ln x-(x-1)}{(x-1)\ln x}$ ← $\frac{0}{0}$ L'Hospital

$= \lim_{x\to1^+} \frac{x\cdot\frac{1}{x}+\ln x-1}{(x-1)\cdot\frac{1}{x}+\ln x}$

$= \lim_{x\to1^+} \frac{\ln x}{1-\frac{1}{x}+\ln x}$ ← $\frac{0}{0}$ L'Hospital

$= \lim_{x\to1^+} \frac{\left(\frac{1}{x}\right)}{\frac{1}{x^2}+\frac{1}{x}}$

$= \lim_{x\to1^+} \frac{x}{1+x}$

$= \frac{1}{1+1}$

$= \boxed{\frac{1}{2}}$

l) $\lim_{x\to 0^+}\left(-\frac{1}{\ln x}\right)^x$

m) $\lim_{x\to 0^+} x^{\sqrt{x}}$ ← 0^0

$\lim_{x\to0^+} \ln x^{\sqrt{x}} = \lim_{x\to0^+} \sqrt{x}\ln x$ ← $0\cdot\infty$

$= \lim_{x\to0^+} \frac{\ln x}{x^{-1/2}}$ ← $\frac{\infty}{\infty}$ L'Hosp

$= \lim_{x\to0^+} \frac{\left(\frac{1}{x}\right)}{-\frac{1}{2}x^{-3/2}}$

$= \lim_{x\to0^+} \frac{-2x^{3/2}}{x}$

$= \lim_{x\to0^+} -2x^{1/2} = 0$

So,

$\lim_{x\to0^+} x^{\sqrt{x}} = \lim_{x\to0^+} e^{\ln x^{\sqrt{x}}}$

$= e^0$

$= \boxed{1}$

n) $\lim_{x\to\infty} (x-1)e^{-x^2}$

o) $\lim_{x\to\infty} x^{1/x}$ $\leftarrow \infty^0$

$$\lim_{x\to\infty}\left(\ln x^{1/x}\right) = \lim_{x\to\infty} \frac{1}{x}\ln x \quad \leftarrow 0\cdot\infty$$

$$= \lim_{x\to\infty} \frac{\ln x}{x} \quad \leftarrow \frac{\infty}{\infty}$$

L'Hosp.

$$= \lim_{x\to\infty} \frac{1/x}{1}$$

$$= 0$$

So,

$$\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} e^{\ln x^{1/x}} = e^0 = \boxed{1}$$

p) $\lim_{x\to 0^+} (1-2x)^{1/x}$

q) $\lim_{x\to\infty}(1+x^2)^{1/x}$ ← ∞^0

$$\lim_{x\to\infty} \ln(1+x^2)^{1/x} = \lim_{x\to\infty} \frac{\ln(1+x^2)}{x} \leftarrow \frac{\infty}{\infty}$$

L'Hospital

$$= \lim_{x\to\infty} \frac{\left(\frac{2x}{1+x^2}\right)}{1}$$

$$= \lim_{x\to\infty} \frac{2x}{1+x^2} \leftarrow \frac{\infty}{\infty}$$

L'Hospital

$$= \lim_{x\to\infty} \frac{2}{2x}$$

$$= 0$$

So, $\lim_{x\to\infty}(1+x^2)^{1/x}$

$$= \lim_{x\to\infty} e^{\ln(1+x^2)^{1/x}}$$

$$= e^0$$

$$= \boxed{1}$$

Q: Solve: TPMWFE

<u>Optional exercises</u> from the Stewart textbook if you'd like more practice:
4.4 (p.311) #9-67 odd