Applications: Newton's Method

(covers Stewart 4.8)

Newton's method is a way of approximating solutions to $f(x) = 0$ using tangent lines of f.

Newton's Method

1. Guess a first approximation to a solution of the equation $f(x) = 0$. This will be an x-value.
2. Use the first approximation to get a second, the second to get a third, and so on, using this:

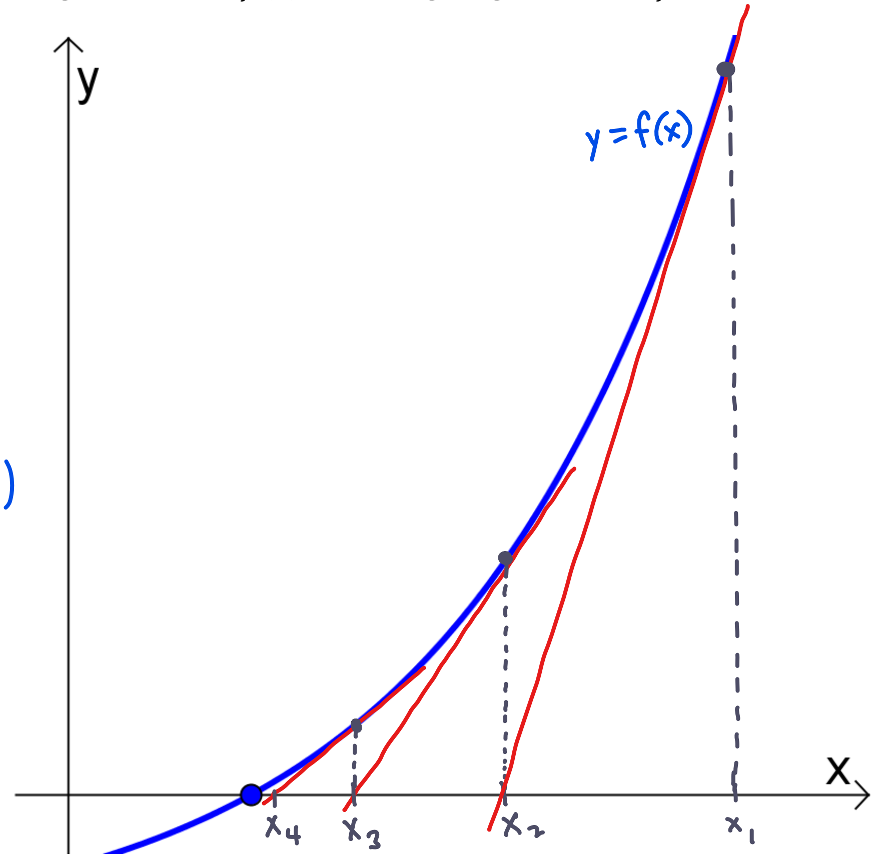
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Why? $L(x) = f(x_1) + f'(x_1)(x - x_1)$

x-int (set y=0): $f(x_1) + f'(x_1)(x - x_1) = 0$

$$x - x_1 = \frac{-f(x_1)}{f'(x_1)}$$

This is x_2 → $x = x_1 - \frac{f(x_1)}{f'(x_1)}$



Ex 1.
Use Newton's method to estimate the positive solution of the equation $\underbrace{x^2 - 2}_{f(x)} = 0$.
Start with $x_1 = 1$ and then find x_4.

$f(x) = x^2 - 2$

$f'(x) = 2x$

$x_1 = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1^2 - 2}{2(1)} = 1 - \left(\frac{-1}{2}\right) = \frac{3}{2} = 1.5$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = \frac{3}{2} - \frac{f(3/2)}{f'(3/2)} = \frac{17}{12} \approx 1.41667$$

$$x_4 = x_3 - \frac{f(x_3)}{f(x_3)} = \frac{17}{12} - \frac{f(17/12)}{f'(17/12)} = \boxed{\frac{577}{408}} \approx 1.41422$$

Note:
$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2} \approx \pm 1.414214$
True zeros

Many scientific calculators have a button called **Ans**, which can save you time. **Ans** refers to the current answer shown on the calculator.

For example, here's how you could use a calculator to do Ex 1:

1, = (This stores 1 as the "answer." In other words, the Ans=1.)

Ans, -, (, Ans, x^2, -, 2,), $\div$, (, 2, $\times$, Ans,), = (Notice that we replaced x_1 with Ans everywhere. After you press =, your screen should show 1.5. So now Ans=1.5.)

= (This reruns the last command with the new value for Ans, 1.5. Now your screen should show approximately 1.41667. So now Ans=1.41667.)

= (You should get about 1.41422.)

Note: If you're using Newton's method with a trig function, be sure that your calculator is in **radians mode**! For example, if you had to estimate the solutions of $\sin x = x^2$.