Applications: Newton's Method

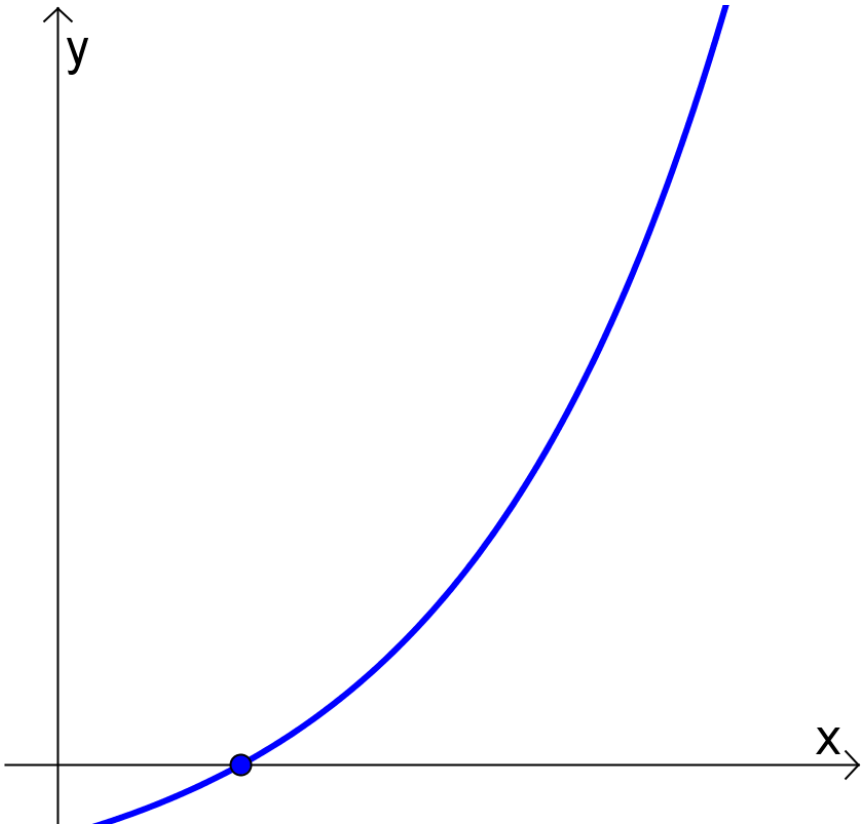
(covers Stewart 4.8)

Newton's method is a way of approximating solutions to $f(x) = 0$ using tangent lines of f.

Newton's Method

1. Guess a first approximation to a solution of the equation $f(x) = 0$. This will be an x-value.
2. Use the first approximation to get a second, the second to get a third, and so on, using this:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



Ex 1.

Use Newton's method to estimate the positive solution of the equation $x^2 - 2 = 0$.
Start with $x_1 = 1$ and then find x_4.

Many scientific calculators have a button called **Ans**, which can save you time. **Ans** refers to the current answer shown on the calculator.

For example, here's how you could use a calculator to do Ex 1:

1, = (This stores 1 as the "answer." In other words, the Ans=1.)

Ans, -, (, Ans, x^2, -, 2,), ÷, (, 2, ×, Ans,), = (Notice that we replaced x_1 with Ans everywhere. After you press =, your screen should show 1.5. So now Ans=1.5.)

= (This reruns the last command with the new value for Ans, 1.5. Now your screen should show approximately 1.41667. So now Ans=1.41667.)

= (You should get about 1.41422.)

Note: If you're using Newton's method with a trig function, be sure that your calculator is in **radians mode**! For example, if you had to estimate the solutions of $\sin x = x^2$.