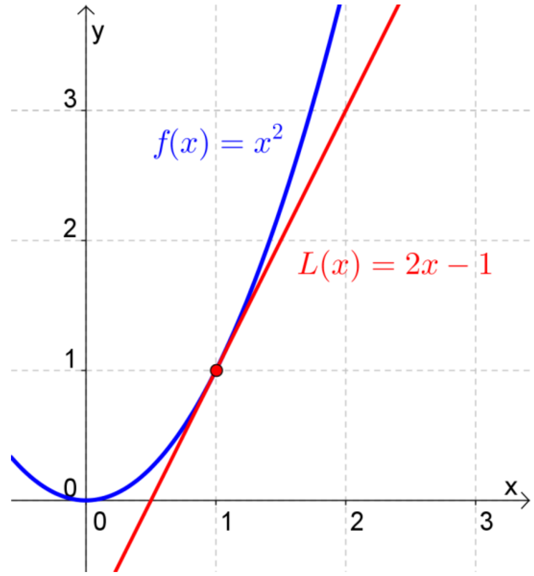
Linear Approximations and Differentials

(covers Stewart 3.10)



We can approximate functions with tangent lines when close to the point of tangency. Since linear functions are relatively easy to work with, this can really help simplify calculations.

Suppose we want to find the tangent line at $x = a$. Then we have a point $(a, f(a))$ on the line, and the slope $f'(a)$ of the line, so using point-slope form, we get:
$y - f(a) = f'(a)(x - a)$

Using function notation, we get what's called the

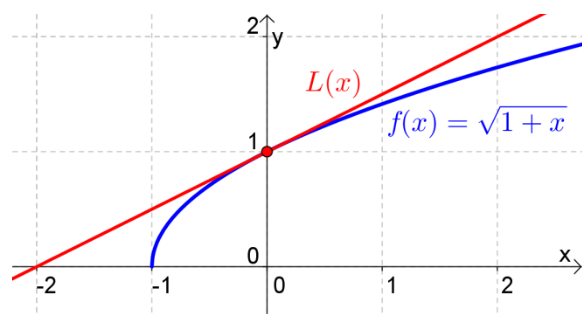
__________________________ of f at a:

$$\boxed{\boldsymbol{L(x) = f(a) + f'(a)(x - a)}}$$

Note that when x is close to a, $f(x) \approx L(x)$.

Ex 1.

Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.



Approximation	True value	Error
$\sqrt{1.2} = \sqrt{1 + 0.2} \approx 1 + \frac{0.2}{2} = 1.10$	1.095445	0.004555
$\sqrt{1.05} = \sqrt{1 + 0.05} \approx 1 + \frac{0.05}{2} = 1.025$	1.024595	0.000405
$\sqrt{1.005} = \sqrt{1 + 0.005} \approx 1 + \frac{0.005}{2} = 1.00250$	1.002497	0.000003

Linearization help approximate a function, but when we want to approximate the *change* in a function, we use what are called

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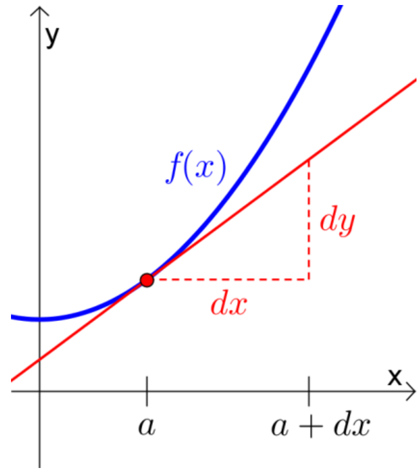
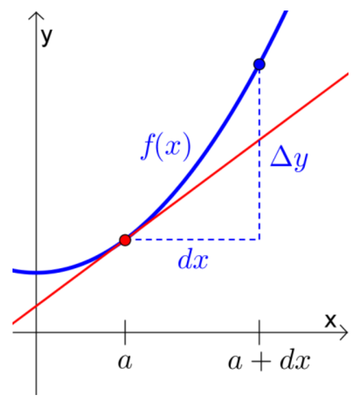


For $y = f(x)$, the **differential dx** is an independent variable, and the **differential dy** is defined as: $\boxed{dy = f'(x)\,dx}$

Ex 2.
Find dy if $y = x^5 + 37x$. Then find dy when $x = 1$ and $dx = 0.2$.



Now find the true change of the function, Δy.

Now calculate the approximation error, $|\Delta y - dy|$.

Ex 3.
The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.05 cm. Use differentials to estimate the maximum possible error in using this value of the radius to compute the volume of the sphere.