**Due date:** ____________ **Name:** ________________________________

1. Find the linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$.

$f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

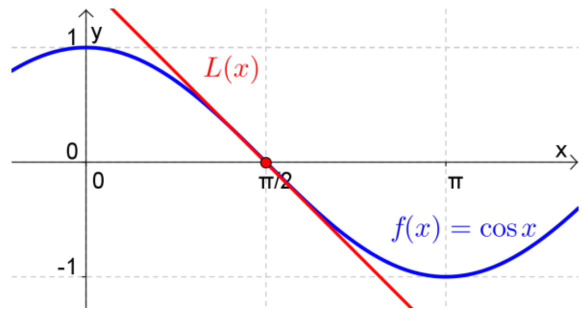
$f'(x) = -\sin x$

$f'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$

$L(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$

$= 0 + (-1)\left(x - \frac{\pi}{2}\right)$

$= \boxed{\frac{\pi}{2} - x}$




2. Find the linearization of $f(x) = e^x$ at $x = 2$.

3. Find the differential ($dy$) of $y = \sin(x^2)$. Then find $dy$ when $x = 1$ and $dx = 0.1$.

$\frac{dy}{dx} = \cos(x^2) \cdot 2x$

$\boxed{dy = 2x\cos(x^2)\,dx}$

$dy = 2(1)\cos(1^2)(0.1)$

$\approx \boxed{0.108}$

4. Find the differential ($dy$) of $y = \ln(x^2 + 1)$. Then find $dy$ when $x = 2$ and $dx = 0.01$.

5. Use a linear approximation to estimate $\sqrt{100.2}$. (Hint: First find the linearization of $f(x) = \sqrt{x}$ at $x = 100$.)

Let $f(x) = \sqrt{x}$

Then $f'(x) = \frac{1}{2\sqrt{x}}$

$$L(x) = f(100) + f'(100)(x-100)$$
$$= \sqrt{100} + \frac{1}{2\sqrt{100}}(x-100)$$
$$= 10 + \frac{1}{20}(x-100)$$
$$= \frac{1}{20}x + 5$$

$$\sqrt{100.2} \approx L(100.2)$$
$$= \frac{1}{20}(100.2) + 5$$
$$= \boxed{10.01}$$

6. Use a linear approximation to estimate $\frac{1}{(3.99)^2}$ to 5 decimal places. Use $a = 4$.

7. Use a linear approximation to estimate $\sqrt[4]{15.99}$ to 5 decimal places. Use $a = 16$.

$f(x) = \sqrt[4]{x}$ $\qquad f(16) = \sqrt[4]{16} = 2$

$f'(x) = \frac{1}{4\sqrt[4]{x^3}}$ $\qquad f'(16) = \frac{1}{4\sqrt[4]{16^3}} = \frac{1}{32}$

$$L(x) = 2 + \frac{1}{32}(x-16)$$

$$\sqrt[4]{15.99} = f(15.99) \approx L(15.99) = 2 + \frac{1}{32}(15.99 - 16)$$

$$\sqrt[4]{15.99} \approx \boxed{1.99969}$$

8. Use a linear approximation to estimate $\sqrt{4.01}$. Use $a = 4$.

9. The radius of a sphere was measured to be 2 inches with a possible error of 0.01 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere. What is the percentage error? Be sure to write units for your answer.

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$

$= 4\pi(2)^2(0.01)$

$= 0.16\pi$

$\frac{dV}{V} = \frac{0.16\pi}{\frac{4}{3}\pi(2)^3} = 0.015$

$0.16\pi \text{ in}^3$

1.5%

Estimated max error: $\boxed{0.16\pi \text{ in}^3}$

Estimated % error: $\boxed{1.5\%}$

10. The radius of a sphere was measured to be 4 inches with a possible error of 0.01 inches. Use differentials to estimate the maximum error in the calculated surface area of the sphere. What is the percentage error? Be sure to write units for your answer.

11. The edge of a cube was found to be 20 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error in computing the surface area of the cube. What is the percentage error? Be sure to write units for your answer.

$S = 6x^2$

$dS = 12x\,dx$

$= 12(20)(0.1)$

$= 24$

$\frac{dS}{S} = \frac{24}{6(20)^2} = \frac{1}{100} = 0.01$

$24 \text{ cm}^2$

1%

Estimated max error: $\boxed{24 \text{ cm}^2}$

Estimated % error: $\boxed{1\%}$

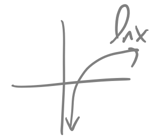
**Review**

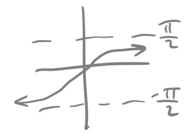
12. Find the following limits.

a) $\lim\limits_{x\to 0^+} \frac{1}{\tan^{-1}(\ln x)} = \boxed{-\frac{2}{\pi}}$

$\ln x \to -\infty$

$\tan^{-1}(\ln x) \to -\frac{\pi}{2}$




b) $\lim\limits_{x\to 2^+} \frac{3x+5}{2-x} = \boxed{-\infty}$

$\frac{\text{top} \to 11}{\text{bot} \to 0,\ \text{neg}}$

c) $\lim\limits_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right)\frac{\left(2x+\sqrt{4x^2+1}\right)}{1\cdot\left(2x+\sqrt{4x^2+1}\right)} = \lim\limits_{x\to\infty} \frac{4x^2-(4x^2+1)}{2x+\sqrt{4x^2+1}} = \lim\limits_{x\to\infty} \frac{-1}{2x+\sqrt{4x^2+1}}$

$\frac{\text{top} \to -1}{\text{bot} \to \infty}$

$= \boxed{0}$

13. Write the limit definition of what it means for a function $f(x)$ to be continuous at $x = a$.

$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

Q: This is an unusual paragraph. I'm curious how quickly you can find out what is so unusual about it. It looks so plain you would think nothing was wrong with it. In fact, nothing is wrong with it! It is unusual though. Study it, and think about it, but you still may not find anything odd. But if you work at it a bit, you might find out. Try to do so without any coaching!

It's missing "e" - one of the most common letters of the alphabet!

<u>Optional exercises</u> from the Stewart textbook if you'd like more practice:
3.10 (p.256) #1, 3, 11-17 odd, 23-27 odd, 33, 35