

Due date: \_\_\_\_\_

Name: \_\_\_\_\_

## Getting Ready for Derivatives (Part 5)

Notes

Recall the following trig identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Also, recall that for logarithms and exponentials:

$$e^{\ln x} = \underline{x} \quad \text{and} \quad \ln e^x = \underline{x}$$

There's also the change of base formula for logarithms:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

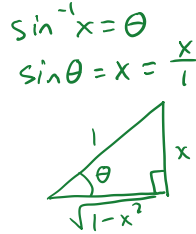
For example, this allows us to convert from any base to natural logarithms:  $\log_b a = \frac{\ln a}{\ln b}$ Lastly, it's time to memorize the definitions of  $\sinh x$  and  $\cosh x$ :

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

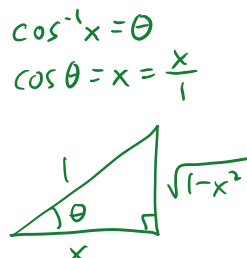
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Notesex: Rewrite  $\cos(\sin^{-1} x)$  as an algebraic expression.

$$\begin{aligned} & \cos(\sin^{-1} x) \\ &= \cos \theta \\ &= \frac{\sqrt{1-x^2}}{1} \\ &= \boxed{\sqrt{1-x^2}} \end{aligned}$$

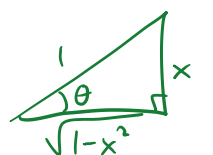
1. Rewrite  $\sin(\cos^{-1} x)$  as an algebraic expression.

$$\begin{aligned} & \sin(\cos^{-1} x) \\ &= \sin \theta \\ &= \frac{\sqrt{1-x^2}}{1} \\ &= \boxed{\sqrt{1-x^2}} \end{aligned}$$



2. Rewrite  $\tan(\sin^{-1} x)$  as an algebraic expression.

$$\begin{aligned} & \tan(\sin^{-1} x) \\ &= \tan \theta \\ &= \frac{x}{\sqrt{1-x^2}} \end{aligned}$$

$$\begin{aligned} \sin^{-1} x &= \theta \\ \sin \theta &= x = \frac{x}{1} \end{aligned}$$


### Practice at home

3. Try to fill in the following from memory.

a)  $\sin^2 x + \cos^2 x = \underline{1}$

b)  $\sin(x + y) = \underline{\sin x \cos y + \cos x \sin y}$

c)  $\cos(x + y) = \underline{\cos x \cos y - \sin x \sin y}$

d)  $e^{\ln x} = \underline{x}$

e)  $\ln e^x = \underline{x}$

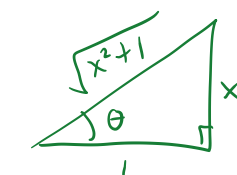
f)  $\log_b a = \frac{\log_c a}{\log_c b}$  (Change of base formula)

g)  $\sinh x = \frac{e^x - e^{-x}}{2}$  (Definition of  $\sinh x$ )

h)  $\cosh x = \frac{e^x + e^{-x}}{2}$  (Definition of  $\cosh x$ )

4. Rewrite  $\sin(\tan^{-1} x)$  as an algebraic expression.

$$\begin{aligned} & \sin(\tan^{-1} x) \\ &= \sin \theta \\ &= \frac{x}{\sqrt{x^2+1}} \end{aligned}$$

$$\begin{aligned} \tan^{-1} x &= \theta \\ \tan \theta &= x = \frac{x}{1} \end{aligned}$$


5. Rewrite  $\cos(\sin^{-1} x)$  as an algebraic expression.

$$\begin{aligned} & \cos(\sin^{-1} x) \\ &= \cos \theta \\ &= \frac{\sqrt{1-x^2}}{1} \\ &= \sqrt{1-x^2} \end{aligned}$$

$$\begin{aligned} \sin^{-1} x &= \theta \\ \sin \theta &= x = \frac{x}{1} \end{aligned}$$
