

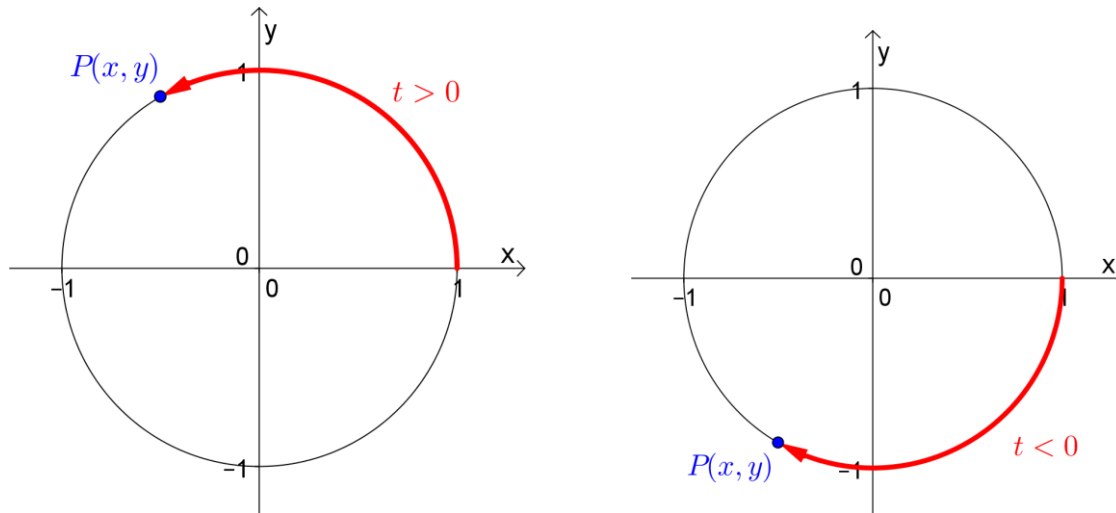
Due date: _____

Name: _____

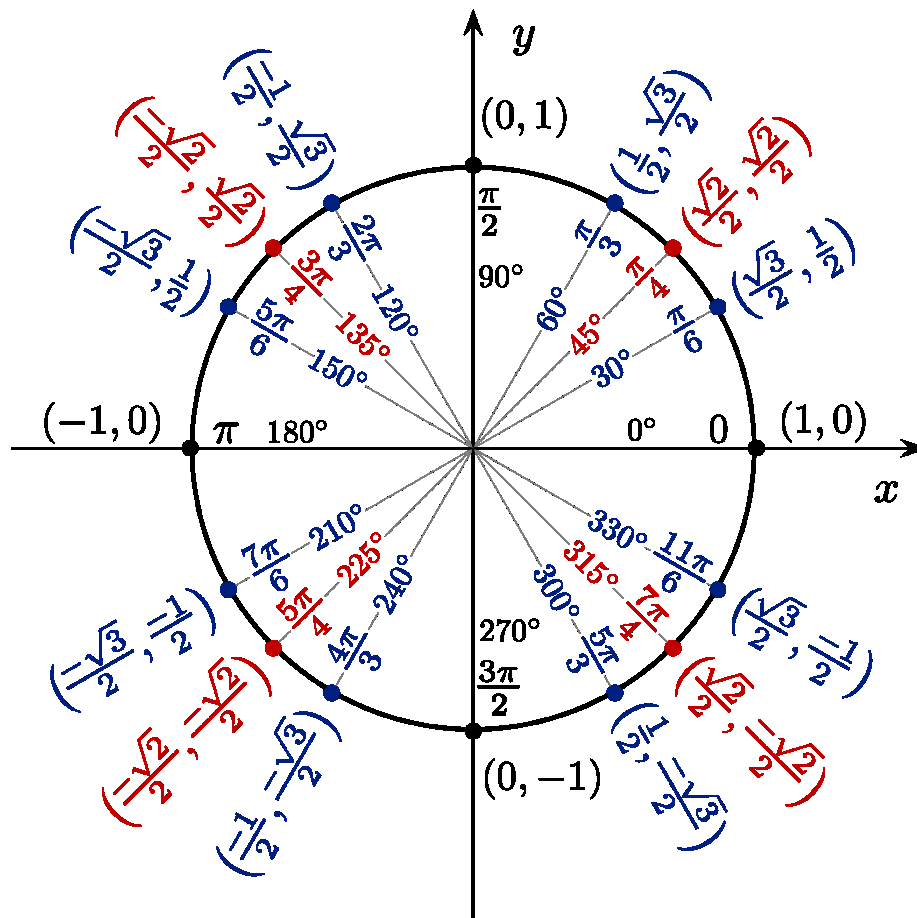
Tools for Limits (Part 3)

If you travel from $(1,0)$ along the unit circle a distance of t , then the point you arrive at $P(x, y)$ is called the **terminal point**.

If t is positive, you travel counterclockwise. If t is negative, you travel clockwise.



Common terminal points on the unit circle:



Recall:

$$\begin{array}{ll} \sin t = y & \csc t = \frac{1}{y} \\ \cos t = x & \sec t = \frac{1}{x} \\ \tan t = \frac{y}{x} & \cot t = \frac{x}{y} \end{array}$$

Notes

ex: $\tan \frac{\pi}{6}$

ex: $\csc \frac{4\pi}{3}$

ex: $\sin\left(-\frac{5\pi}{6}\right)$

1. Find the exact value without a calculator.

a) $\cos \pi$

b) $\sin \frac{\pi}{2}$

c) $\sin \frac{3\pi}{4}$

d) $\sec \frac{5\pi}{6}$

e) $\tan\left(-\frac{\pi}{3}\right)$

f) $\tan\left(\frac{5\pi}{4}\right)$

Notes

ex: Solve: $2 \sin 2x + \sqrt{3} = 0$

Trigonometric Equations

2. Find all solutions to the following equations.

a) $2 - \sin x = 1$

b) $3 \tan x - 1 = -4$

c) $\cos 2x - 1 = 0$

d) $2 \sin 3x - 1 = 0$

Recall some of the key trig identities:

Reciprocal Identities

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x} & \cot x &= \frac{\cos x}{\sin x} \\ \csc x &= \frac{1}{\sin x} & \sec x &= \frac{1}{\cos x} & \cot x &= \frac{1}{\tan x} \end{aligned}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

Addition and Subtraction Formulas

$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y \\ \sin(x - y) &= \sin x \cos y - \cos x \sin y \end{aligned}$$

$$\begin{aligned} \cos(x + y) &= \cos x \cos y - \sin x \sin y \\ \cos(x - y) &= \cos x \cos y + \sin x \sin y \end{aligned}$$

Double-Angle Formulas

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1 \end{aligned}$$

Notes

ex: Solve: $2 + \ln 3x = 5$

Logarithmic and Exponential Equations

3. Solve the following equations.

a) $3 + 2 \log_2 x = 1$

b) $6 - \ln 2x = 10$

c) $3e^{5x+1} + 7 = 10$

Practice at home

4. Find the exact value without a calculator.

a) $\cos \frac{\pi}{2}$

b) $\sin \pi$

c) $\csc \frac{5\pi}{6}$

d) $\tan \frac{3\pi}{2}$

e) $\sec \left(-\frac{2\pi}{3} \right)$

f) $\cot \frac{7\pi}{4}$

5. Find all solutions to the following equations.

a) $2 \cos x - \sqrt{3} = 0$

b) $\sqrt{3} \tan x = -1$

c) $2 \sin 2x + \sqrt{2} = 0$

d) $2 \cos 3x - 1 = 0$

6. Solve the following equations.

a) $1 = 6 + \ln 3x$

b) $2(3 + \log_3 4x) = 8$

7. Solve the following equations.

a) $5 \cdot 2^{3x-4} = 15$

b) $3e^{x+2} - 1 = 4$