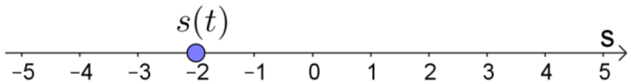
Applications: Rates of Change

(covers Stewart 3.7)

Suppose a particle (i.e. a dot) is moving along a coordinate line, and its position is given by $s(t)$.



Position: $s(t)$ (here, position can also be interpreted as displacement relative to 0)

Velocity: $v(t) = s'(t)$

Acceleration: $a(t) = v'(t) = s''(t)$

Note: Velocity can be positive or negative. Positive means movement to right, negative means movement to left. The speed of the particle, however, is always positive, and is given by $|\boldsymbol{v(t)}|$.

Ex 1.
The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ (where $t \geq 0$ is measured in seconds and s is measured in meters).
Find the velocity at time t.

What is the velocity after 2 seconds?

When is the particle at rest?

When is the particle moving in the positive direction?

Sketch a diagram to represent the motion of the particle.

Find the total distance traveled during the first 5 seconds.

Find the acceleration at time t and after 3 seconds.

When is the particle speeding up? When is it slowing down?

Geometry

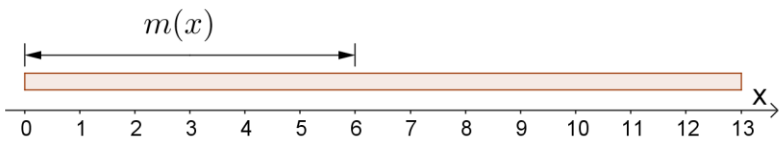
Ex 2.

How fast is the area of a circle changing with respect to the radius when the radius is 5 cm?

Density

Suppose you have a wire or rod of length L and mass m. If the mass is evenly distributed, then the linear density is $\rho = \frac{m}{L}$ (where the units might be something like $\frac{kg}{m}$).

But what if the mass is not evenly distributed? Suppose we have a function for the mass of the wire from 0 to x, call it $m(x)$.



Based on this, how can we determine the linear density at any given point on the wire?

Ex 3.

The mass of a wire from the left end to a point x cm to the right is $\sqrt{x}$ grams. Find the linear density when x is 3 cm.