

The Chain Rule

(covers parts of Stewart 3.4, 3.6, 3.11)

ex: If a car uses 0.2 gallons per mile when travelling 60 miles per hour, how many gallons per hour are being used?

Ex 1.

Differentiate $y = (2x - 5)^2$.

The Chain Rule

If $y = f(u)$ is a differentiable function of u ,
and $u = g(x)$ is a differentiable function of x ,
then $y = f(g(x))$ is a differentiable function of x , and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Here's another way to write the Chain Rule using functional notation:

$$(f \circ g)'(x) = \underbrace{f'(g(x))}_{\text{derivative of outer}} \cdot \underbrace{g'(x)}_{\text{derivative of inner}}$$

Here, think: “ f is the outer function, and g is the inner function”

Let's get some practice identifying inner and outer functions:

	Inner function	Outer function
$(x^2 + 1)^{10}$		
$\frac{1}{e^x + \sin x}$		
$\sqrt{2x^2 + \frac{1}{x}}$		
$\sin(\log x)$		
e^{-x^3}		
$\ln(5x + 2)$		

Ex 2.

Differentiate $\sin(x^2 + e^x)$ with respect to x .

Ex 3.

Find the derivative of $y = e^{\cos x}$.

Ex 4.

Find the derivative of $f(x) = \tanh(5 - \sin 2x)$.

Ex 5.

Find the derivative of $f(x) = \sqrt{\ln(x + 2)}$.

Ex 6.

Find the derivative of $f(x) = \ln(x^3 \cos x)$.

Ex 7.

$$\frac{d}{dx} \left(\frac{1}{3x-2} \right) =$$

Ex 8.

Differentiate $f(x) = (3x + 1)^4(2x - 1)^5$ and simplify your answer by factoring.

Ex 9.

Find all points where $f(x) = \sin^2 x$ has a horizontal tangent line.