Due date: ____________ **Name: ___________________________________**

Note: Write your answers using positive exponents. Radicals are nice, but not required.
ex: Write $\frac{1}{x^2}$ not x^{-2}.
ex: $\sqrt{x}$ is nicer than $x^{1/2}$. (Either $x^{3/2}$ or $x\sqrt{x}$ are fine.)

1. Find the derivative of each of the following functions.

a) $y = -\frac{1}{4}x^3 - \frac{\sqrt{x}}{2} + 7x - 168 + x^{3/2}$

b) $f(x) = \frac{2}{x} + \sqrt[3]{x} - \frac{3}{x^2} - \frac{x^4}{5} + \frac{2}{\sqrt[3]{x}}$

c) $y = 2\csc x - 3\sin x + \frac{\tan x}{2}$

d) $f(x) = \cot x - \sec x + \frac{2}{3}\cos x$

e) $y = 3\cos^{-1} x + \pi - 2\tan^{-1} x + \csc^{-1} x$

f) $f(x) = \frac{2}{5}\tan^{-1} x + 11x - x^{1.1} + \sin^{-1} x$

g) $y = -\frac{e^x}{2} + 3\log_2 x - e^2 + 2 \cdot 5^x$

h) $f(x) = 2^5 + 4 \cdot 3^x - \frac{2\ln x}{3} - 5e^x$

i) $y = -\frac{\sinh x}{7} + 8\tanh x + 6\,\text{sech}\, x$

j) $f(x) = \cosh x + \frac{3}{5}\text{csch}\, x - \coth x$

k) $y = \pi^3 - \frac{5}{\sqrt{e}} + \frac{e^{\pi}}{\sqrt{2-\sqrt{3}}} - \ln 17^{1000^5} + 1000000000000066600000000000001$

(Note: The last number is a palindromic prime number called Belphegor's prime. Notice that it reads the same forwards and backwards, has the number 666 in the middle (The Number of the Beast!), and has an unlucky 13 zeros on each side. So, derivatives ward off devils and bad luck! 😉)

2. Differentiate each of the following functions.
   a) $f(x) = 2x^4 \cos x$

   b) $y = (3x^3 - x)(2 - e^x)$

   c) $f(x) = \ln x \sin x$

   d) $f(x) = 2^x \sin^{-1} x$

   e) $f(x) = \frac{5-x}{x^2-3}$

   f) $y = \frac{x^2+3}{4x-1}$

g) $y = \frac{x}{\cos x}$

h) $f(x) = \frac{\cosh x}{1 - \sinh x}$

i) $y = \frac{\tan x}{x + \csc x}$

j) $f(x) = \frac{x+1}{x^2 e^x}$

k) $y = \frac{e^x \sin x}{x^3}$

l) $y = \frac{3^x \cos x}{2x^2 \sinh x}$
(Note: Don't worry about simplifying this one!)

m) $y = \frac{x \ln x}{\csc x \operatorname{sech} x}$
(Note: Don't worry about simplifying this one!)

n) $y = \frac{\sin x + \cos x + 1}{\sin x}$ (Hint: simplify first!)

o) $y = x \ln x \tan x$

p) $y = e^x \sin x \cos x$

3. Find the second derivative of $y = 4 - 2x + 2\sqrt{x} - x^{-3}$.

4. Find $\frac{d^2y}{dx^2}$ if $y = 3x^{3/2} + 2e^x - 4 \cdot 2^x$.

5. Find $f''(x)$ if $f(x) = \sec x$.

6. Find the second derivative of $y = \frac{3x^2+2x}{\sqrt{x}}$. (Hint: simplify first!)

7. Find the fifth derivative of $f(x) = \frac{2}{x}$.

8. Find an equation for the tangent line to $y = x^3 - 2e^x$ at the point $(0, -2)$.

9. Find an equation for the tangent line to $y = x \sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

10. Find an equation for the tangent line of $y = (\sin x + \cos x) \sec x$ at $\left(\frac{\pi}{4}, 2\right)$. (Hint: simplify first!)

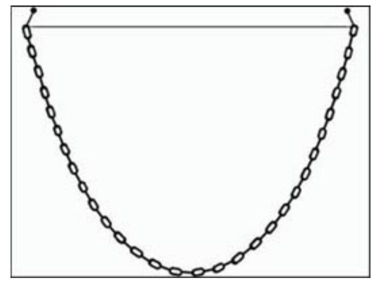
Hyperbolic Functions

The **hyperbolic functions** are:

$\mathbf{sinh}\,x$ $\quad$ $\mathbf{cosh}\,x$ $\quad$ $\mathbf{tanh}\,x$ $\quad$ $\mathbf{csch}\,x$ $\quad$ $\mathbf{sech}\,x$ $\quad$ $\mathbf{coth}\,x$

$\sinh x$ is read either "hyperbolic sine of x" or "sinch of x" (like "Hyperbolic functions are a cinch!")
$\cosh x$ is read either "hyperbolic cosine of x" or "cosh of x" (like "Oh my cosh—another function!")
$\tanh x$ is read either "hyperbolic tangent of x" or "tanch of x"

Whereas trigonometric functions satisfy the identity: $\sin^2 x + \cos^2 x = 1$,
hyperbolic functions satisfy the identity: $\cosh^2 x - \sinh^2 x = 1$.
So, whereas $(\cos x, \sin x)$ is a point on the unit circle, $(\cosh x, \sinh x)$ is a point on a hyperbola.



Hyperbolic functions appear as solutions to certain differential equations, as well as some areas of physics. Interestingly, $\cosh x$ is how you would model the shape of a hanging chain (this shape is called a "catenary," which means "chain" in Latin).

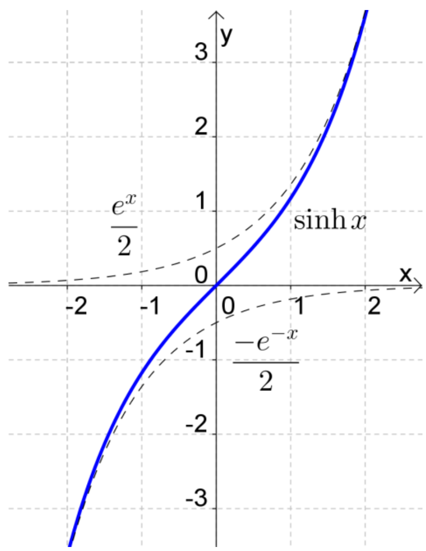
Here are how the hyperbolic functions are defined:

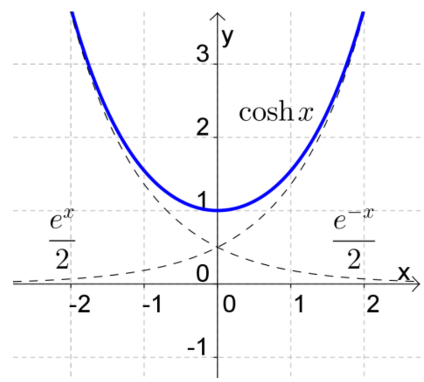
$\sinh x = \frac{e^x - e^{-x}}{2}$ $\qquad$ $\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{\sinh x}{\cosh x}$ $\qquad$ $\coth x = \frac{\cosh x}{\sinh x}$

$\operatorname{sech} x = \frac{1}{\cosh x}$ $\qquad$ $\operatorname{csch} x = \frac{1}{\sinh x}$

The graphs of $\sinh x$ and $\cosh x$ are to the right.





There are also inverse hyperbolic functions, and their derivatives.
$\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$,
$\operatorname{csch}^{-1} x$, $\operatorname{sech}^{-1} x$, $\coth^{-1} x$

It can be shown that these inverse hyperbolic functions can be written in terms of logarithms:

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$ for $x \geq 1$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$

The following homework exercises are meant to help you get to know the hyperbolic functions while reviewing limits.

11. Find the following.
    a. $\lim_{x\to\infty} \sinh x$

    b. $\lim_{x\to-\infty} \sinh x$

    c. $\lim_{x\to\infty} \cosh x$

    d. $\lim_{x\to-\infty} \cosh x$

    e. $\lim_{x\to 0} \cosh x$

    f. $f'(0)$ if $f(x) = \cosh x$

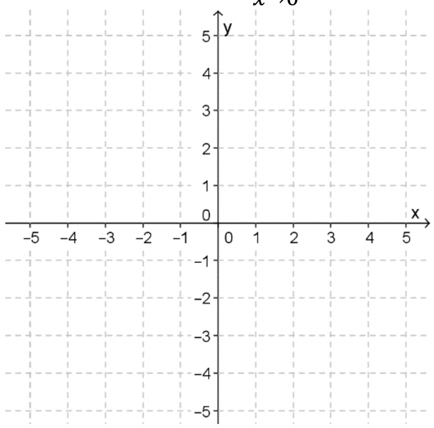
Review

12. Find the following limits.
    a) $\lim_{x\to 1} \ln|x-1|$

    b) $\lim_{x\to\infty} \csc(e^{-x})$

    c) $\lim_{x\to-\infty} \left(3x + \sqrt{x^2+1}\right)$

13. Sketch the graph of a function $f(x)$ that is discontinuous at $x = 1$, continuous from the left at $x = 1$, and satisfies $\lim_{x\to 0} f(x) = 3$, $f(0) = -2$, $\lim_{x\to -\infty} f(x) = 1$, and $\lim_{x\to -2^+} f(x) = \infty$.



Challenge Problem: Find $\frac{d^{875}}{dx^{875}}(\sin x)$.

Q: What are the next two letters in the following series and why?
W A T N T L I T F S _ _

<u>Optional exercises</u> from the Stewart textbook if you'd like more practice:

3.1 (p.180) #3-35 odd, 45

3.2 (p.188) #3-31 odd

3.3 (p.196) #1-15 odd, 21, 23, 25a, 27a, 29

3.6 (p.223) #17, 23