Math 180 - Test #3 Info and Review Exercises

Fall 2017, Prof. Beydler

Test Info

- Date: Thursday, November 30, 2017
- Will cover packets #17 through #25.
- You'll have the entire class to finish the test.
- For this test, you'll need a **scientific calculator**.
- No notes, no books, no phones, no smart watches during the test.
- There will be a seating chart for the test.
- Where to get help as you're studying:
  - Office hours
  - TMARC, LAC, or other tutoring centers
  - E-mail me at dbeydler@mtsac.edu

Formulas and stuff

(Note: Know all of these except for the ones with 🏝 next to them, which I'll give you. This list is not meant to include everything you'll need to know on the test.)

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$(cf)' = cf'$

$(f \pm g)' = f' \pm g'$

$(fg)' = fg' + gf'$

$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ 🏝

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ 🏝

$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$ 🏝

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ 🏝

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ 🏝

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$ 🏝

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ 🏝

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ 🏝

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ 🏝

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ 🏝

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ 🏝

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$ 🏝

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\tanh x = \frac{\sinh x}{\cosh x}$

$\coth x = \frac{\cosh x}{\sinh x}$ $\operatorname{sech} x = \frac{1}{\cosh x}$ $\operatorname{csch} x = \frac{1}{\sinh x}$

$\cosh^2 x - \sinh^2 x = 1$

Here are some helpful geometry formulas to know for optimization (4.7) problems:

Pythagorean Theorem: $a^2 + b^2 = c^2$ (or $(leg)^2 + (leg)^2 = (hypotenuse)^2$)
Area of rectangle: $A = lw$
Area of circle: $A = \pi r^2$
Area of triangle: $A = \frac{1}{2}bh$
Circumference of circle: $C = 2\pi r = \pi d$
How to get perimeter of any polygon (just add the lengths of the sides).
How to get the surface area of a 3-D surface (just add the areas of the faces/sides).
Volume of a box (also called a rectangular prism): $V = lwh$
Volume of circular cylinder: $V = \pi r^2 h$
Surface area of sphere: $S = 4\pi r^2$
Volume of sphere: $V = \frac{4}{3}\pi r^3$
Volume of cone: $V = \frac{1}{3}\pi r^2 h$

Summation Properties

$$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} ca_i = c \cdot \sum_{i=1}^{n} a_i \qquad \sum_{i=1}^{n} c = n \cdot c$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Integral Properties

$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
$\int_a^a f(x)\,dx = 0$
$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
$\int_a^b \big(f(x) \pm g(x)\big)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
If $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.
The Fundamental Theorem of Calculus, Part 1: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
The Fundamental Theorem of Calculus, Part 2: $\int_a^b f(x)\,dx = F(b) - F(a)$
Substitution: $\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du$
Integration by parts: $\int u\,dv = uv - \int v\,du$

Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes, practice problems, previous quizzes, and homework problems to fully prepare for the test.

1. Find the absolute maximum and minimum values of $f(x) = x - \ln(x^2)$ on the interval $[1,3]$.

2. Find the absolute maximum and minimum values of $f(x) = xe^{-2x}$ on the interval $[-2, 2]$.

3. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 9 - x^2$. What are the dimensions of such a rectangle that maximize its area?

4. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5. If you send a package via USPS Express Mail International, the length (the longest dimension) plus the girth (distance around thickest part) can't be more than 79 inches. What's the largest possible volume of a rectangular box with a square base that can be sent by USPS Express Mail International? And what are the dimensions of the box?

6. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.

7. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

8. An open box with a square base is going to be made. The sides of the box will cost $3 per square foot, and the base will cost $5 per square foot. What are the dimensions of the box with the largest volume that can be constructed for $60?

9. Find the most general antiderivative for each of the following functions.
   a) $f(x) = 8\sqrt[3]{x} - \frac{2}{x} + x^{9999} - \frac{10}{\sqrt{x}} + \frac{1}{x^3} + x\sqrt{x}$
   b) $f(x) = \sin 3x + 2\sec x \tan x - e^{-x} + 3^x + 19$
   c) $f(x) = \frac{2}{1+x^2} - \frac{3}{\sqrt{1-x^2}} - \csc^2 x$

10. A particle is moving with the given data. Find the position of the particle.
    $a(t) = 3\cos t - 2\sin t,\ s(0) = 0,\ v(0) = 4$

11. A particle is moving with the given data. Find the position of the particle.
    $a(t) = t^2 - 4t + 6,\ s(0) = 0,\ s(1) = 20$

12. Find f if $f''(x) = 2e^x + 3\sin x$, $f(0) = 0$, and $f(\pi) = 0$.

13. Estimate the area under the graph of $f(x) = \sqrt{x}$ between $x = 0$ and $x = 4$ using…
    a) …a lower sum with four rectangles of equal width.
    b) …an upper sum with four rectangles of equal width.
    c) …midpoints with four rectangles of equal width.

14. Estimate the area under the graph of $f(x) = e^{-x} + 1$ between $x = -1$ and $x = 1$ using…
   a) …right endpoints with four rectangles of equal width.
   b) …left endpoints with four rectangles of equal width.
   c) …midpoints with four rectangles of equal width.

15. Estimate the distance traveled in 12 hours given the following sample velocities…

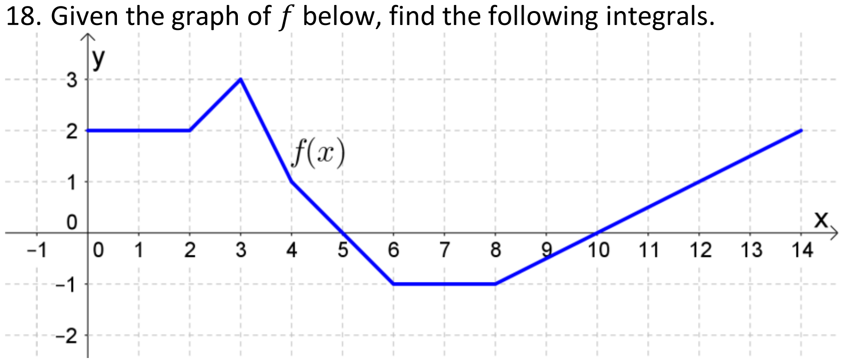
Time (hours)	0	2	4	6	8	10	12
Velocity (mph)	60	63	64	50	45	48	70

   a) …using left-endpoint values.
   b) …using right-endpoint values.

16. Evaluate the following integrals <u>using Riemann sums with right endpoints</u>.
   a) $\int_0^3 (2x + x^2)\,dx$
   b) $\int_0^4 (x^2 - 4x + 2)\,dx$

17. Suppose that $\int_1^5 f(x)\,dx = -2$, $\int_7^1 f(x)\,dx = 10$, and $\int_1^5 g(x)\,dx = 3$. Find the following.
   a) $\int_5^1 3f(x)\,dx$
   b) $\int_1^5 [g(x) - 4f(x)]\,dx$
   c) $\int_5^7 f(x)\,dx$

18. Given the graph of f below, find the following integrals.



   a) $\int_0^1 f(x)\,dx$
   b) $\int_0^3 f(x)\,dx$
   c) $\int_3^6 f(x)\,dx$
   d) $\int_4^6 f(x)\,dx$
   e) $\int_4^{10} f(x)\,dx$
   f) $\int_0^{14} f(x)\,dx$

19. Graph the following integrands and use the area under the graph to evaluate the integral.
   a) $\int_{-1}^6 (|x-3| + 1)\,dx$
   b) $\int_0^2 \left(3 + \sqrt{4 - x^2}\right) dx$

20. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the following functions.
   a) $f(x) = \int_3^{1/x} \sin^5 t\,dt$
   b) $g(x) = \int_x^3 (t^2 - 5)^{10}\,dt$
   c) $h(x) = \int_{\sqrt{x}}^{2x} \tan^{-1} t\,dt$

21. Evaluate the following integrals.
    a) $\int_4^9 \left(2x\sqrt{x} - 3\right) dx$
    b) $\int_0^{\pi/8} \sin 2x \, dx$
    c) $\int_{\pi/6}^{\pi/2} \csc^2 x \, dx$
    d) $\int_0^1 \frac{5}{1+x^2} dx$
    e) $\int_0^{\pi/3} (2x - \sec x \tan x) \, dx$
    f) $\int_{1/2}^1 \frac{1}{2x} dx$
    g) $\int_0^1 e^{3x} \, dx$
    h) $\int_0^7 \frac{1}{\sqrt[3]{x+1}} dx$
    i) $\int \frac{4x^2+2}{(2x^3+3x)^4} dx$
    j) $\int e^{2\cos x} \cdot \sin x \, dx$
    k) $\int 3xe^{x^2+2} \, dx$
    l) $\int x^2 e^{-2x} \, dx$
    m) $\int x^2 \ln x \, dx$
    n) $\int e^{2x} \cos x \, dx$
    o) $\int x\sqrt{2x+1} \, dx$
    p) $\int_e^{e^2} \frac{(\ln x)^3}{2x} dx$
    q) $\int_0^{\pi/2} (7^{\cos x} \sin x) \, dx$
    r) $\int_0^1 \tan^{-1} x \, dx$
    s) $\int_{-3}^3 |x| \tan x \, dx$

22. Suppose the velocity function of a particle is $v(t) = t^2 - 2t$ (in meters per second). Find the distance traveled by the particle during the time period $-2 \le t \le 5$.