

Test #2 Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes, practice problems, previous quizzes, and homework problems to fully prepare for the test.

1. Find $\frac{dy}{dx}$.

a) $x^2 \sin y - \frac{3x}{y} = x^5$

b) $x^2 e^y = \sqrt{y}$

c) $\cot(x + y) = 1 - \ln(y^2 + 3)$

2. Find an equation for the tangent line at the given point.

a) $x^2 + 4xy + y^2 = 13$, $(2, 1)$

b) $y = x^2 \cos^{-1}(3x + 3)$, $(-1, 1)$

3. Use logarithmic differentiation to find $\frac{dy}{dx}$.

a) $y = \frac{e^{-5x^2+1} \cdot \sqrt[3]{x^2-4} \cdot \sin x}{x \cdot (x^3+1)^5 \cdot \sqrt{x+5}}$

b) $y = x^{\sec x}$

c) $y = (\csc x)^{1/x}$

4. The position of a particle is given by the equation $s(t) = t^3 - 12t + 3$ (where $t \geq 0$ is measured in seconds and s is measured in meters).

a) What is the velocity after 1 second?

b) When is the particle at rest?

c) When is the particle moving in the positive direction?

d) Sketch a diagram to represent the motion of the particle.

e) Find the total distance traveled during the first 3 seconds.

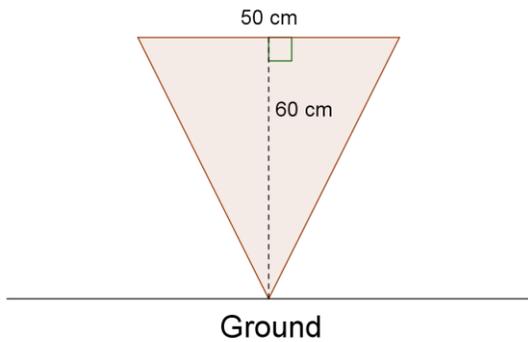
f) Find the acceleration at time t and after 5 seconds.

g) When is the particle speeding up? When is it slowing down?

5. How fast is the volume of a sphere changing with respect to the radius when the radius is 3 inches?
6. The mass of a thin rod from the left end to a point x mm to the right is $3 \ln(x + 2)$ grams. Find the linear density when x is 5 mm.
7. The width of a rectangle is increasing at a rate of 5 cm/s and its length is decreasing by 1 cm/s.
- When the width is 12 cm and the length is 16 cm, is the area of the rectangle increasing or decreasing? How fast is the area increasing or decreasing?
 - Is the perimeter of the rectangle increasing or decreasing? How fast is the perimeter increasing or decreasing?
8. A ladder 20-ft long rests against a vertical wall. If the bottom of the ladder slides away at 2 ft/s, how fast is the top sliding down the wall when the bottom is 12 ft from the wall?

9. At 3pm, plane A is 50 miles north of plane B. Plane A is flying east at 120 mph and plane B is flying west at 80 mph. How fast is the distance between the planes changing at 5pm?

10. A trough is 5 meters long, and has cross sections that are isosceles triangles with base 50 cm and height 60 cm (as shown below). If the trough is being filled with water at a rate of $300 \text{ cm}^3/\text{min}$, how fast is the water level rising when the water is 20 cm deep?



11. Prove that the derivative of $y = \cot x$ is $\frac{dy}{dx} = -\csc^2 x$ by using the derivatives of $\sin x$ and $\cos x$.

12. Prove that the derivative of $y = \ln x$ is $\frac{dy}{dx} = \frac{1}{x}$.

13. Prove that the derivative of $y = \cos^{-1} x$ is $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$.

14. Find the differential (dy) of $y = e^{-x} \cos x$.

15. Find the differential (dy) of $y = \sec^{-1}(3x)$.

16. Use a linear approximation (or differentials) to estimate $\frac{1}{(2.999)^5}$.

17. The radius of a circle was measured to be 5 ft with a possible error of 0.1 ft.

a) Use differentials to estimate the maximum error in the calculated area of the circle. What is the relative error? What is the percentage error?

b) Use differentials to estimate the maximum error in the calculated circumference of the circle. What is the relative error? What is the percentage error?

18. Use Newton's method to estimate the positive root of $\frac{1}{x} = 1 + x^3$ correct to six decimal places. Start with $x_1 = 0.8$.

19. Find the following limits.

a) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$

b) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2 \sin x}{3x^3}$

c) $\lim_{x \rightarrow \infty} x(e^{1/x} - 1)$

d) $\lim_{x \rightarrow 0^+} \sin x \ln(\sin x)$

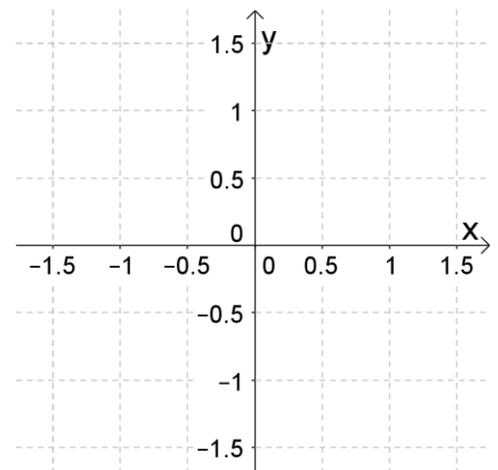
e) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

f) $\lim_{x \rightarrow 0^+} x^{x^2}$

g) $\lim_{x \rightarrow \infty} (x + 1)^{e^{-x}}$

20. Let $f(x) = \frac{x}{x^2+1}$.

- a) Find the domain of f .
- b) Find the x -intercept(s) and y -intercept of f (if any).
- c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).
- d) Find f' and f'' , and determine where each are 0 and/or do not exist (DNE).
- e) Do a sign analysis on f' and f'' .
- f) Find the intervals on which f is increasing and decreasing.
- g) Find the intervals on which f is concave up and concave down.
- h) Find all local maxima, local minima, and inflection points of f .
- i) Sketch the graph of f .



22. Let $f(x) = 4x^{1/3} - x^{4/3}$.

- a) Find the domain of f .
- b) Find the x -intercept(s) and y -intercept of f (if any).
- c) Find vertical asymptote(s) and horizontal asymptote(s) (if any).
- d) Find f' and f'' , and determine where each are 0 and/or do not exist (DNE).
- e) Do a sign analysis on f' and f'' .
- f) Find the intervals on which f is increasing and decreasing.
- g) Find the intervals on which f is concave up and concave down.
- h) Find all local maxima, local minima, and inflection points of f .
- i) Sketch the graph of f .

