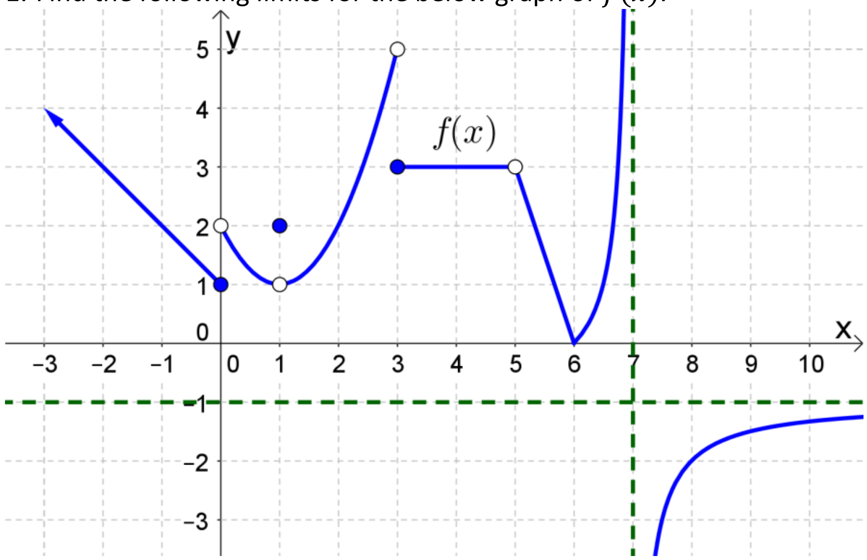
Test #1 Review Exercises

Note: If you write up the answers to all of the review exercises listed below, and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes, practice problems, previous quizzes, and homework problems to fully prepare for the test.

1. Find the following limits for the below graph of $f(x)$.



$\lim_{x\to 0^-} f(x)$	$\lim_{x\to 0^+} f(x)$	$\lim_{x\to 0} f(x)$
$\lim_{x\to 1^-} f(x)$	$\lim_{x\to 1^+} f(x)$	$\lim_{x\to 1} f(x)$
$\lim_{x\to 2^-} f(x)$	$\lim_{x\to 2^+} f(x)$	$\lim_{x\to 2} f(x)$
$\lim_{x\to 3^-} f(x)$	$\lim_{x\to 3^+} f(x)$	$\lim_{x\to 3} f(x)$
$\lim_{x\to 5^-} f(x)$	$\lim_{x\to 5^+} f(x)$	$\lim_{x\to 5} f(x)$
$\lim_{x\to 6^-} f(x)$	$\lim_{x\to 6^+} f(x)$	$\lim_{x\to 6} f(x)$
$\lim_{x\to 7^-} f(x)$	$\lim_{x\to 7^+} f(x)$	$\lim_{x\to 7} f(x)$
$\lim_{x\to \infty} f(x)$	$\lim_{x\to -\infty} f(x)$	

2. Using the graph of $f(x)$ from the previous question, answer the following.
   a) Is f continuous or discontinuous at $x = 1$? Why?
   b) Is f continuous or discontinuous at $x = 5$? Why?
   c) Is f continuous or discontinuous at $x = 6$? Why?
   d) Is f continuous from the left at $x = 1$? Why or why not?
   e) Is f continuous from the right at $x = 3$? Why or why not?
   f) Is f continuous from the left at $x = 7$? Why or why not?
   g) Is f differentiable at $x = 6$?
   h) Is f differentiable at $x = -2$?
   i) Find $f'(-5)$.
   j) Find $f'(1)$.
   k) Find $f'(3)$.
   l) Find $f'(4)$.
   m) Find $f'(5.5)$.
   n) Find an equation of the tangent line at $x = -2$.

3. Find the following limits.

a) $\lim_{x \to -1} \frac{x^2-2x-3}{x^2-1}$

b) $\lim_{x \to -5^+} \frac{3-x}{x+5}$

c) $\lim_{x \to -2} \frac{3x+4}{x+2}$

d) $\lim_{x \to 2} \frac{3x^2-5x-2}{x^2-5x+6}$

e) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

f) $\lim_{x \to 1^-} \frac{x-\sqrt{x}}{x-1}$

g) $\lim_{x \to 0} \frac{\frac{1}{x-3}+\frac{1}{3}}{x(x-1)}$

h) $\lim_{x \to \infty} \tan^{-1}(2-x^2)$

i) $\lim_{x \to 0^+} e^{2-\ln x}$

j) $\lim_{x \to 1^-} \tan\left(e^{\csc(x-1)}\right)$

k) $\lim_{x \to -\infty} \frac{2x^5 - x + 1}{7x^5 + 4x^4 - 3}$

l) $\lim_{x \to \infty} \frac{e^x - 2}{1 + e^{2x}}$

m) $\lim_{x \to -\infty} \frac{x^3 - 21x}{5x + 2}$

n) $\lim_{x \to \infty} \frac{x + 2}{\sqrt{3x^2 - 5x}}$

o) $\lim_{x \to -\infty} \left(2x + \sqrt{4x^2 + 1}\right)$

p) $\lim_{x \to 0^+} (\cot x - \ln x)$

4. Use the Squeeze Theorem to prove that $\lim_{x\to 1}(x-1)^2 \sin\left(\frac{5}{x-1}+3\right)=0$.

5. Explain why $f(x)=\begin{cases} 2x^2-3 & \text{if } x\le -2 \\ \frac{5}{x+2} & \text{if } x>-2 \end{cases}$ is discontinuous at $x=-2$.

6. Explain why $f(x)=\begin{cases} x^2+2 & \text{if } x<1 \\ \sqrt{x+8} & \text{if } x>1 \end{cases}$ is discontinuous at $x=1$.

7. How would you define $f(-1)$ in a way that makes $f(x)=\frac{5+4x-x^2}{2x^2-x-3}$ continuous at $x=-1$?

8. Find all values of c such that $f(x) = \begin{cases} x^2 - c & \text{if } x \leq 2 \\ \sin\frac{\pi}{4}x & \text{if } x > 2 \end{cases}$ is continuous on $(-\infty, \infty)$.

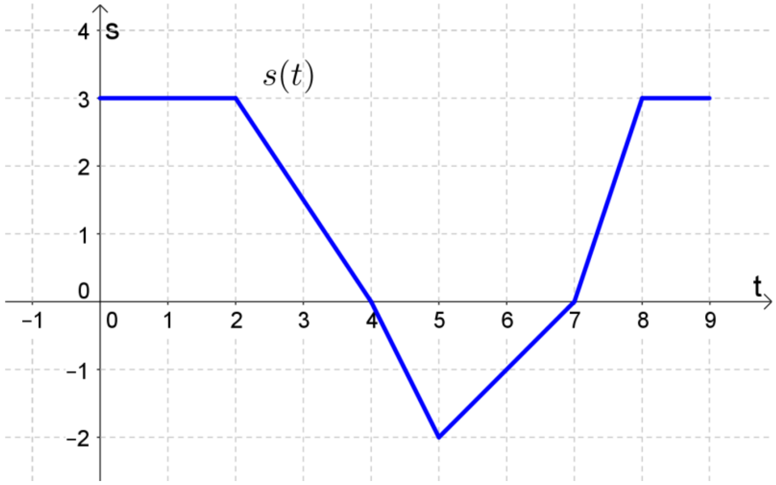
9. True or false: If $\lim_{x \to 3^-} f(x) = -\infty$, then f has a vertical asymptote $x = 3$.

10. True or false: A function is continuous at $x = a$ if $\lim_{h \to 0} f(x) = f(a)$.

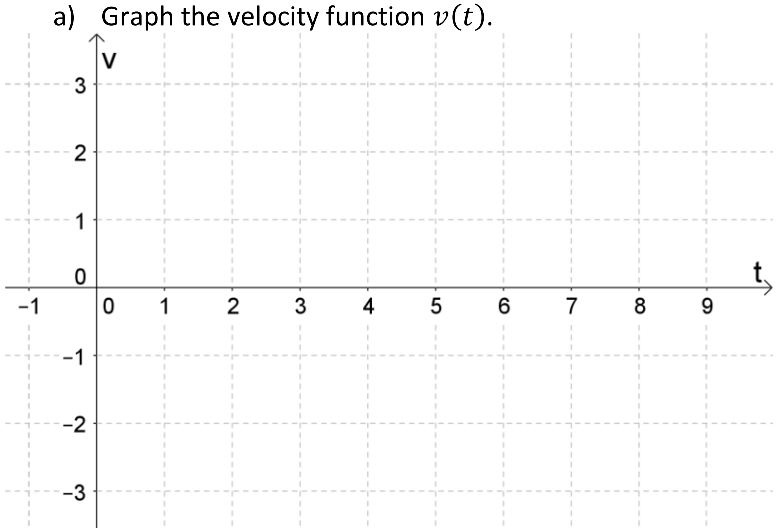
11. Use the Intermediate Value Theorem to show that there is a root of $x - 3 = \sin 2x$ in the interval $\left(\frac{\pi}{2}, \pi\right)$.

12. Sketch the graph of an example of a function f that satisfies $\lim_{x \to -3} f(x) = -\infty$, $\lim_{x \to 4^-} f(x) = -\infty$, $\lim_{x \to 4^+} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 2$, and $f(1) = 0$.

13. Suppose an ant moves back and forth along a line, and that the position of the ant over time is given by the function $s(t)$ below. Use meters for s and seconds for t.



a) Graph the velocity function $v(t)$.



b) Find $s'(3)$.

c) Find $s'(5)$.

d) Find $v(7.5)$.

14. Find the derivatives of the following functions <u>using the limit definition</u>.

a) $f(x) = 2x^2 - 3x$

b) $f(x) = \sqrt{3x+1}$

c) $f(x) = \frac{2}{x+5}$

15. Differentiate the following functions.

a) $f(x) = 3\pi^5 - 6\sqrt[3]{x^2} + \frac{2}{x^4} - 2x^{1.3} + 5e^2 + \frac{4}{3\sqrt{x}} - 17e^x$

b) $f(x) = \frac{x^4 - x\sqrt{x} + 2}{\sqrt{x}}$

c) $f(x) = x^3 e^x + \frac{2}{x}$

d) $f(x) = \frac{e^x+2}{x^2-x+3}$

e) $f(x) = 3\cos x - 2x^3 \sin x$

f) $f(x) = \sec x \csc x$

g) $f(x) = \frac{\cot x - 3x^2}{\tan x}$

h) $f(x) = \frac{e^x \sin x}{x^3}$

i) $f(x) = \ln\left(\sin\sqrt{2x+3}\right)$

j) $y = 3^{x^2 - \cos(\log_2 x)}$

k) $f(x) = \left(\ln\sqrt{x} + x^5\right)\tan(e^{3x+1})$

l) $y = \frac{\sin^3 x}{\ln(x^2+3)}$

m) $f(x) = \frac{x^2+1}{\tan^{-1} x}$

n) $y = \sqrt[3]{x^2 \sin^{-1} x}$

o) $f(x) = \cosh(\sinh(\tanh x))$

16. Find an equation for the tangent line at the given point.

a) $y = x^3 - 2e^x,\ (0, -2)$

b) $y = \tan x,\ \left(\frac{\pi}{4}, 1\right)$

c) $y = 2\sin x\cos x - x^2,\ (0, 0)$

17. Find the second derivatives of the following functions.

a) $y = 4e^x + 2x^3 - x + \frac{1}{x}$

b) $y = x^2\cos x$