_____ / 15 total points

Quiz #2

Name: _____

Math 180, Section 7, Prof. Beydler

Thursday, October 12, 2017

Directions: Show all work. No books or notes. A scientific calculator is allowed. Your desk and lap must be clear (no phones, notebooks, etc.). Write your answers in the indicated places, or box your answers. Good luck!

1. (4 points) Find an equation of the tangent line to $2y - 1 = e^{xy}$ at $(0, 1)$.

Answer: $y = \frac{1}{2}x + 1$

$$\frac{d}{dx}(2y) - \frac{d}{dx}(1) = \frac{d}{dx}(e^{xy})$$

$$2\frac{dy}{dx} = e^{xy} \cdot \left(x \cdot \frac{dy}{dx} + 1 \cdot y\right)$$

$$2\frac{dy}{dx} = xe^{xy}\frac{dy}{dx} + ye^{xy}$$

$$2\frac{dy}{dx} - xe^{xy}\frac{dy}{dx} = ye^{xy}$$

$$\frac{dy}{dx}(2 - xe^{xy}) = ye^{xy}$$

$$\frac{dy}{dx} = \frac{ye^{xy}}{2 - xe^{xy}}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1 \cdot e^{0 \cdot 1}}{2 - 0 \cdot e^{0 \cdot 1}} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x + 1$$

2. (3 points) Use <u>logarithmic differentiation</u> to find the derivative of y with respect to x.

$y = x^{\sin x}$

$\frac{dy}{dx} =$ $y\left(\frac{\sin x}{x} + \cos x \ln x\right)$

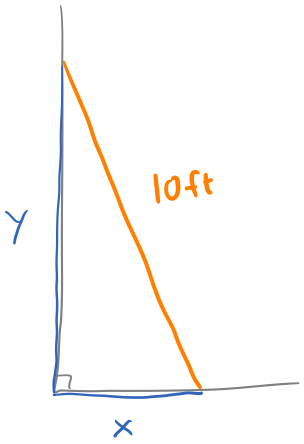
$$\ln y = \ln x^{\sin x}$$

$$= \sin x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \cos x \cdot \ln x$$

$$\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \ln x\right)$$

3. (4 points) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away at 3 ft/s, how fast is the top sliding down the wall when the bottom is 2 ft from the wall? Be sure to write the units for your answer.



$x^2+y^2=10^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$\frac{dy}{dt}=\frac{-x\frac{dx}{dt}}{y}$

$=\frac{-2(3)}{\sqrt{96}}$

$\approx -0.612\ ft/s$

When $x=2$:

$2^2+y^2=10^2$

$y=\sqrt{96}$

Answer: $0.612\ ft/s$

4. The position of a particle is given by the equation $s(t)=t^3-6t^2+9t$ (where $t\geq 0$ is measured in seconds and s is measured in meters).

a) (1 point) When is the particle at rest?

$v(t)=3t^2-12t+9=3(t^2-4t+3)=3(t-1)(t-3)$

$t=1$ $t=3$

Answer: $t=1, 3$ sec

b) (1 point) Find the total distance traveled during the first 5 seconds. Be sure to include units for your answer.

$|s(1)-s(0)|=|4-0|=4$

$|s(3)-s(1)|=|0-4|=4$

$|s(5)-s(3)|=|20-0|=20$

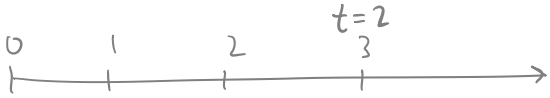
} Add

Answer: 28 m

c) (2 points) When is the particle speeding up? When is it slowing down?

$a(t)=6t-12=6(t-2)$

$t=2$



$v(t)$	+	0	−	−	−	0	+
$a(t)$	−	−	−	0	+	+	+

Speeding up: $1<t<2,\ t>3$

Slowing down: $0\leq t\leq 1,\ 2<t<3$