Derivative Shortcuts

(covers parts of Stewart 3.1, 3.3, 3.5, 3.6, and 3.11)

Remember these derivatives!

These three are so common you'll soon take them for granted. Here, c is a constant.

$\frac{d}{dx}(c) = 0$ $\quad$ $\frac{d}{dx}(cx) = c$ $\quad$ $\frac{d}{dx}(x^n) = nx^{n-1}$

Constant Multiple: $(cf)' = cf'$

Sum/Difference Rules: $(f \pm g)' = f' \pm g'$

Product Rule: $(fg)' = fg' + f'g$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Psst! Here are the trig derivatives. Notice the functions that start with "c" have negative derivatives.

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$ $\quad$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Memorize the following three inverse trig derivatives. I'll give you $\csc^{-1} x$, $\sec^{-1} x$, and $\cot^{-1} x$.

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

In Calculus, base e is exceptionally excellent!

$\frac{d}{dx}(e^x) = e^x$ $\quad$ $\frac{d}{dx}(a^x) = a^x \ln a$

$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0)$ $\quad$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

These are called hyperbolic functions. You'll learn more about these in the homework.

$\frac{d}{dx}(\sinh x) = \cosh x$ $\quad$ $\frac{d}{dx}(\cosh x) = \sinh x$ $\quad$ $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

Don't worry about memorizing the following. I'll give them to you on quizzes and tests.

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\quad$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\quad$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\quad$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\quad$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

Ex 1.

Find the following derivatives.

$\frac{d}{dx}(-15) =$

$\frac{d}{dx}(11x) =$

$\frac{d}{dx}(x^3) =$

$\frac{d}{dx}\left(\frac{1}{x}\right) =$

$\frac{d}{dx}(x^{\pi+e}) =$

$\frac{d}{dx}(\sqrt{x}) =$

Ex 2.

Find the derivative of $y = x^3 + \frac{4}{3}x^2 - 5x + 1$.

Ex 3.

Find the derivative of $y = 4\sqrt{x} - \frac{2}{3x}$.

Ex 4.

Find the derivative of $y = 2\cos x - \sec x + 3\cot x$.

Ex 5.

Find the derivative of $f(x) = -3\sin^{-1} x + \frac{2}{3}\tan^{-1} x$.

Ex 6.

Find the derivative of $y = 4\ln x + \frac{e^x}{2} - 5^x + \frac{1}{2}\log_2 x$.

Ex 7.

Find the derivative of $f(x) = -\cosh x - 7\tanh x$.

Product Rule: $\boldsymbol{\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + \frac{d}{dx}[f(x)]g(x)}$

In other words, $\boldsymbol{(fg)' = fg' + f'g}$

Ex 8.

Differentiate $y = \frac{1}{x}(x^2 + e^x)$.

Ex 9.

Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

In other words, $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

Ex 10.

Differentiate $y = \frac{x^2 - 1}{x^3 + 1}$.

The Second Derivative

$f''(x) = \frac{d}{dx}[f'(x)]$ also written $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or $y'' = (y')'$

Ex 11.

Find the second derivative of $y = 2x^4 - 5x^2 + 23x - 10$

Note: If $s(t)$ is a position function, then $s'(t)$ is the velocity function, and $s''(t)$ is the acceleration function. Acceleration measures the rate of change of velocity.

The nth Derivative

To get the third derivative, fourth derivative, fifth derivative, etc., just keep differentiating.
For example, here are the derivatives of $f(x) = 4x^3 - 2x^2 + 5x - 1$:

First derivative: $f'(x) = 12x^2 - 4x + 5$
Second derivative: $f''(x) = 24x - 4$
Third derivative: $f'''(x) = 24$
Fourth derivative: $f^{(4)}(x) = 0$
Fifth derivative: $f^{(5)}(x) = 0$