Integration by Parts

(covers Stewart 7.1)

Let's derive what's called the Integration by Parts formula:

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule for derivatives)

$f(x)g(x) = \int [f(x)g'(x) + g(x)f'(x)]\, dx$

$\int [f(x)g'(x) + g(x)f'(x)]\, dx = f(x)g(x)$

$\int f(x)g'(x)\, dx + \int g(x)f'(x)\, dx = f(x)g(x)$

$\int f(x)g'(x)\, dx = f(x)g(x) - \int g(x)f'(x)\, dx$

Rewrite using $u = f(x)$ and $v = g(x)$, so that $du = f'(x)\, dx$ and $dv = g'(x)\, dx$ to get the **Integration by Parts formula**:

$$\boxed{\int \boldsymbol{u\, dv = uv - \int v\, du}}$$

Let's see how to use it…

Ex 1.

$\int x \sin x\, dx$

Note that if we had chosen u and dv differently, it wouldn't work:

The goal is to turn an integral $\int u\,dv$ that you *can't* do into an integral $\int v\,du$ that you *can* do.

Ex 2.

$\int \ln x\ dx$

Ex 3.

$\int x^2 e^x\,dx$

Here's a quick way to do integration by parts multiple times, called **tabular integration**:

Ex 4.

$\int x^2 e^x\,dx$

Ex 5.

$\int x^3 \sin 2x \, dx$

Sometimes when doing integration by parts, you'll get back to the original integral.

Ex 6.

$\int e^x \cos x \, dx$