

Due date: _____

Name: _____

1. Find the absolute maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval $[0.4, 2]$.

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = \frac{(x+1)(x-1)}{x^2}$$

$$f' = 0:$$

$$(x+1)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \cancel{x = -1} & x = 1 \end{array}$$

Not in
[0.4, 2]

$$f' \text{ DNE:}$$

$$x^2 = 0$$

$$\cancel{x = 0}$$

Not in [0.4, 2]

x	f(x)
1	2 ← Abs min
0.4	2.9 ← Abs. max
2	2.5

2. Find the absolute maximum and minimum values of $f(x) = 2x^{\frac{4}{3}} - x^{\frac{1}{3}}$ on the interval $[-1, 1]$.

3. Find the absolute maximum and minimum values of $f(x) = x - \ln x$ on the interval $\left[\frac{1}{2}, 2\right]$.

$$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$$

$$\begin{aligned} f' &= 0: \\ x-1 &= 0 \\ x &= 1 \end{aligned}$$

f' DNE:
 ~~$x=0$~~
 Not in interval
 or domain

x	$f(x)$
1	1 ← Abs. min
$\frac{1}{2}$	$\frac{1}{2} - \ln \frac{1}{2} \approx 1.193$
2	$2 - \ln 2 \approx 1.307$ ← Abs. max

4. Find the absolute maximum and absolute minimum values of $f(x) = \sqrt{9-x^2}$ on $[-3, 2]$.

5. Find the absolute maximum and minimum values of $f(x) = \frac{x}{1+x^2}$ on the interval $[0,4]$.

$$f'(x) = \frac{(1+x^2) - x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

$f'=0:$
 $1-x^2=0$
 $(1-x)(1+x)=0$

↓ ↓
 $x=1$ ~~$x=-1$~~
 not in
 interval
 $[0,4]$

f' DNE:
 $(1+x^2)^2=0$
 Nowhere

x	$f(x)$
1	$\frac{1}{2} \leftarrow \text{Abs. max}$
0	$0 \leftarrow \text{Abs. min}$
4	$\frac{4}{17}$

6. Find the absolute maximum and absolute minimum values of $f(x) = x + \cot\left(\frac{x}{2}\right)$ on $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$.

Q: What has three feet but no legs or arms? *A yard stick!*

Optional exercises from the Stewart textbook if you'd like more practice:

4.1 (p.283) #3, 5, 47-61 odd