Graphing using Calculus

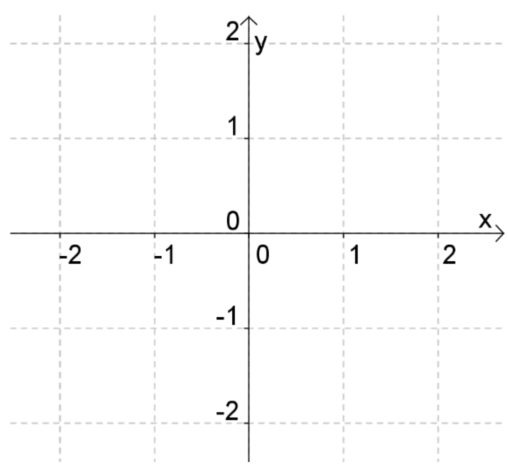
(covers Stewart 4.5)

How to sketch a curve (using the techniques we know so far):

1. Find **domain**.
2. Find/plot **intercepts**. (if possible)
3. Find/draw **asymptotes**.
4. Find $\boldsymbol{f'}$ and $\boldsymbol{f''}$, and determine when each are **0** or **DNE**.
5. Do a **sign analysis** on f' and f''.
6. Find/plot **maxima/minima**, and **inflection points**.
7. Sketch curve.

Ex 1.

Sketch the graph of $f(x) = 3x^4 + 4x^3$



Review of asymptotes

1. A **vertical asymptote** $x = a$ happens when $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.

For rational functions $\frac{P(x)}{Q(x)}$, they occur when $Q(x) = 0$ and $P(x) \neq 0$.

ex: Find the vertical asymptotes of $f(x) = \frac{x^3}{x^4 - 16}$.

ex: Does $f(x) = x \ln x$ have a vertical asymptote at $x = 0$?

2. A **horizontal asymptote** $y = L$ happens when $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

For rational functions $\frac{P(x)}{Q(x)}$ …

…if degree(P) < degree(Q), then the horizontal asymptote is $y = 0$.

…if degree(P) = degree(Q), then the horizontal asymptote is $y = \frac{\text{degree}(P)}{\text{degree}(Q)}$.

…if degree(P) > degree(Q), then no horizontal asymptotes.

ex: Find the horizontal asymptote of $f(x) = \frac{5x^2}{3x^2 + 7}$.

ex: Find the horizontal asymptote of $f(x) = x^3 e^{-x}$.

Ex 2.

Sketch the graph of $f(x) = \frac{x}{(x+1)^2}$