_____ / 50 total points

Test #2 Name: _____

Math 180, Section 5, Prof. Beydler Thursday, October 26, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (4 points) Find an equation for the tangent line of $x + 3xy = 2 + \cos y$ at $(3, 0)$.

Answer: _____

2. (3 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ given that $y = (\cos x)^{1/x}$.

Answer: _____

3. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ (where $t \geq 0$ is measured in seconds and s is measured in meters).

   a) (1 point) When is the particle at rest?

   Answer: _____

   b) (1 point) When is the particle moving in the positive direction?

   Answer: _____

   c) (2 points) Find the total distance traveled during the first 4 seconds. Be sure to include units for your answer.

   Answer: _____

   d) (1 point) Find the acceleration after 2 seconds. Be sure to include units for your answer.

   Answer: _____

   e) (3 points) When is the particle speeding up? When is it slowing down?

   Speeding up: _____

   Slowing down: _____

4. (1 point) The mass of a thin wire from the left end to a point x inches to the right is $\sqrt{3x+1}$ ounces. Find the linear density when x is 5 inches. Be sure to write units for your answer.

   Answer: _____

5. (4 points) At 1:30pm, car A is 10 miles west of car B. Car A is moving south at 30 mph and car B is moving north at 40 mph. How fast is the distance between the cars changing at 3:30pm? Be sure to write units for your answer.

Answer: _____

6. (3 points) Prove that the derivative of $y = \tan x$ is $\frac{dy}{dx} = \sec^2 x$ by using the derivatives of $\sin x$ and/or $\cos x$.

7. (3 points) Use a linear approximation (or differentials) to estimate $\sqrt[3]{7.99}$ to 5 decimal places. Be sure to show your work.

$\sqrt[3]{7.99} \approx$ _____

8. (3 points) The radius of a sphere was measured to be 5 inches with a possible error of 0.03 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere. What is the percentage error? Be sure to write units for your answer.

Estimated maximum error: _____

Estimated percentage error: _____

9. Find the following limits. Be sure to show your work.

a) (3 points) $\lim_{x\to 0^+} \frac{e^x-1}{x\cos x}$

Answer: _____

b) (3 points) $\lim_{x\to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

Answer: _____

10. Let $f(x) = xe^x$.

a) (1 point) Find the x-intercept(s) and y-intercept of f (if none, write "none").

x-intercept(s): _____ y-intercept: _____

b) (1 point) Find the horizontal asymptote of f.

Horizontal asymptote: _____

c) (4 points) Find f' and f'', and determine where each are 0 and/or do not exist (DNE). If nowhere, write "nowhere."

$f'(x) = 0$ when $x =$ _____

$f'(x)$ DNE when $x =$ _____

$f''(x) = 0$ when $x =$ _____

$f''(x)$ DNE when $x =$ _____

d) (2 points) Do a sign analysis on f' and f''.

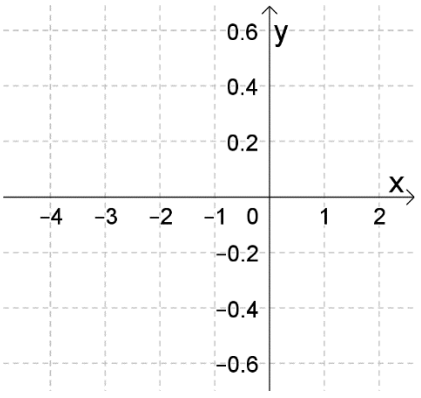
e) (1 point) Find the intervals on which f is increasing and decreasing.

Increasing: _____ Decreasing: _____

f) (1 point) Find the intervals on which f is concave up and concave down.

Concave up: _____ Concave down: _____

g) (1 point) Find all local maxima, local minima, and inflection points of f (if any). Be sure to identify which is a max/min/inflection point.



h) (2 points) Sketch the graph of f.

11. (2 points) Use Newton's method to estimate the positive root of $\sin x = x^2$ correct to six decimal places. Start with $x_1 = 1$.

Answer: _____

Note: Be sure to double check your work. And remember to turn in your homework and extra credit! ☺

Here are some formulas that I promised to give:

$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$ $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$