

## Test #1 Review Exercise Answers

1.

$$\lim_{x \rightarrow 0^-} f(x) = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = 2$$

$$\lim_{x \rightarrow 3^-} f(x) = 5$$

$$\lim_{x \rightarrow 5^-} f(x) = 3$$

$$\lim_{x \rightarrow 6^-} f(x) = 0$$

$$\lim_{x \rightarrow 7^-} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -1$$

$$\lim_{x \rightarrow 0^+} f(x) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = 2$$

$$\lim_{x \rightarrow 3^+} f(x) = 3$$

$$\lim_{x \rightarrow 5^+} f(x) = 3$$

$$\lim_{x \rightarrow 6^+} f(x) = 0$$

$$\lim_{x \rightarrow 7^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow 0} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$\lim_{x \rightarrow 2} f(x) = 2$$

$$\lim_{x \rightarrow 3} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow 5} f(x) = 3$$

$$\lim_{x \rightarrow 6} f(x) = 0$$

$$\lim_{x \rightarrow 7} f(x) \text{ DNE}$$

2.

a) discontinuous because  $\lim_{x \rightarrow 1} f(x) = 1$  but  $f(1) = 2$  (so  $\lim_{x \rightarrow 1} f(x) \neq f(1)$ )

b) discontinuous because  $f(5)$  is undefined

c) continuous because  $\lim_{x \rightarrow 6} f(x) = 0 = f(6)$

d) no because  $\lim_{x \rightarrow 1^-} f(x) = 1$  but  $f(1) = 2$  (so  $\lim_{x \rightarrow 1^-} f(x) \neq f(1)$ )

e) yes because  $\lim_{x \rightarrow 3^+} f(x) = 3 = f(3)$

f) no because  $f(7)$  is undefined (or you could say "because  $\lim_{x \rightarrow 7^-} f(x) = \infty$ ")

g) no

h) yes

i)  $-1$

j) DNE

k) DNE

l)  $0$

m)  $-3$

n)  $y - 3 = -(x + 2)$  (or  $y = -x + 1$ )

3.

a)  $2$

b)  $\infty$

c) DNE

d)  $-7$

e)  $\frac{1}{6}$

f)  $\frac{1}{2}$

g)  $\frac{1}{9}$

h)  $-\frac{\pi}{2}$

i)  $\infty$

j)  $0$

k)  $\frac{2}{7}$

l)  $0$

m)  $\infty$

n)  $\frac{1}{\sqrt{3}}$

o)  $0$

p)  $\infty$

4. First off,  $-1 \leq \sin\left(\frac{5}{x-1} + 3\right) \leq 1$ . Multiplying by  $(x-1)^2$ , we have  $-(x-1)^2 \leq \sin\left(\frac{5}{x-1} + 3\right) \leq (x-1)^2$ . Since  $\lim_{x \rightarrow 1} (-(x-1)^2) = \lim_{x \rightarrow 1} (x-1)^2 = 0$ , by the Squeeze Theorem  $\lim_{x \rightarrow 1} (x-1)^2 \sin\left(\frac{5}{x-1} + 3\right) = 0$ . ■

5.  $\lim_{x \rightarrow -2^-} (2x^2 - 3) = 5$  but  $\lim_{x \rightarrow -2^+} \frac{5}{x+2} = \infty$ , so  $\lim_{x \rightarrow -2} f(x)$  DNE. Thus,  $f$  is discontinuous at  $x = -2$ .

6.  $f(1)$  is undefined. Thus,  $f$  is discontinuous at  $x = 1$ .

7.  $-\frac{6}{5}$

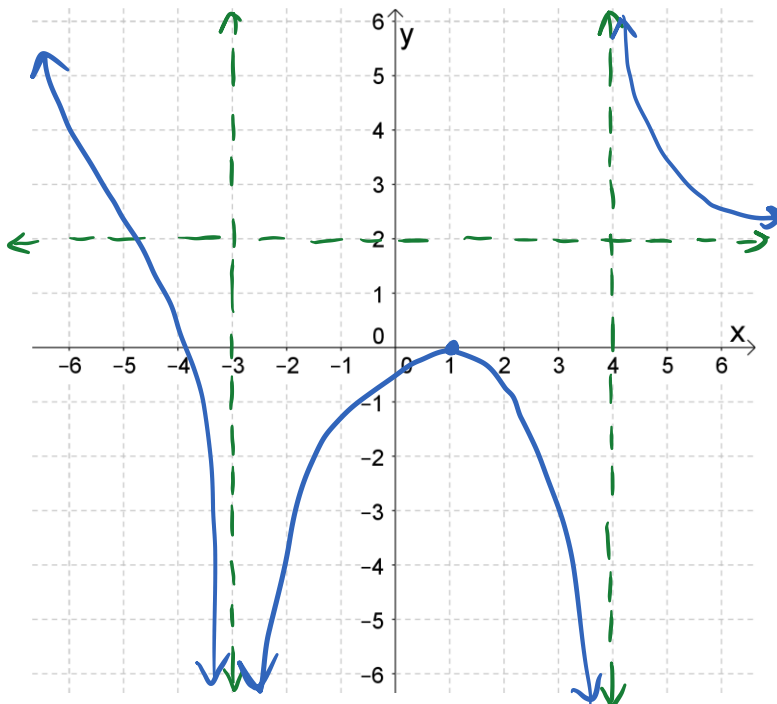
8.  $c = 3$

9. True

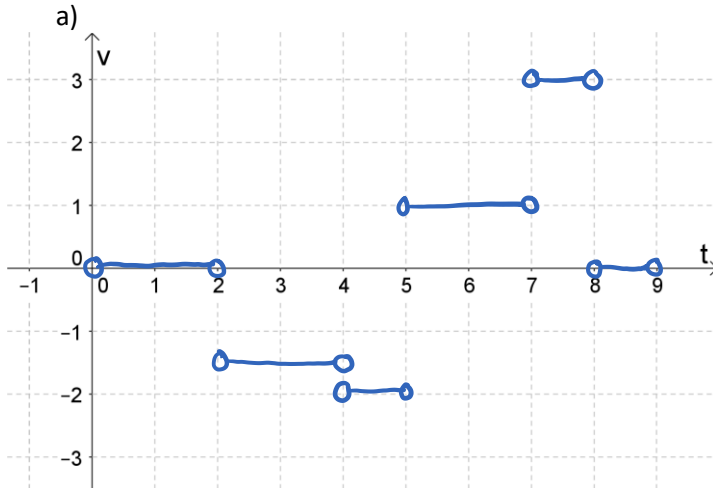
10. False (It should be  $\lim_{x \rightarrow a} f(x) = f(a)$ .)

11. First, subtract the  $\sin 2x$  to get  $x - 3 - \sin 2x = 0$ . Then let  $f(x) = x - 3 - \sin 2x$ . Since  $f$  is continuous,  $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} - 3 < 0$ , and  $f(\pi) = \pi - 3 > 0$ , we know by the Intermediate Value Theorem that there must be some  $x$  in the interval  $\left(\frac{\pi}{2}, \pi\right)$  such that  $f(x) = 0$ . At that  $x$ , we have a root of  $x - 3 = \sin 2x$ . ■

12.



13.



- b)  $-1.5$  m/s  
 c) DNE  
 d)  $3$  m/s

14.

- a)  $4x - 3$   
 b)  $\frac{3}{2\sqrt{3x+1}}$   
 c)  $-\frac{2}{(x+5)^2}$

15. False (The theorem goes: If  $f$  is differentiable at  $x = a$ , then  $f$  is continuous at  $x = a$ .)

16.

- a)  $f'(x) = -\frac{4}{\sqrt[3]{x}} - \frac{8}{x^5} - 2.6x^{0.3} - \frac{2}{3x\sqrt{x}} - 17e^x$   
 b)  $f'(x) = \frac{7}{2}x^2\sqrt{x} - 1 - \frac{1}{x\sqrt{x}}$   
 c)  $f'(x) = x^3e^x + 3x^2e^x - \frac{2}{x^2}$   
 d)  $f'(x) = \frac{x^2e^x - 3xe^x + 4e^x - 4x + 2}{(x^2 - x + 3)^2}$   
 e)  $f'(x) = -3\sin x - 2x^3\cos x - 6x^2\sin x$   
 f)  $f'(x) = -\sec x \csc x \cot x + \csc x \sec x \tan x$   
 g)  $f'(x) = \frac{-\tan x \csc^2 x - 6x \tan x - \cot x \sec^2 x + 3x^2 \sec^2 x}{\tan^2 x}$   
 h)  $f'(x) = \frac{xe^x \cos x + xe^x \sin x - 3e^x \sin x}{x^4}$  (or  $\frac{e^x(x \cos x + x \sin x - 3 \sin x)}{x^4}$ )  
 i)  $\frac{\cos \sqrt{2x+3}}{\sqrt{2x+3} \sin \sqrt{2x+3}}$  (or  $\frac{\cot \sqrt{2x+3}}{\sqrt{2x+3}}$ )  
 j)  $3^{x^2 - \cos(\log_2 x)} \cdot \ln 3 \cdot \left(2x + \sin(\log_2 x) \cdot \frac{1}{x \ln 2}\right)$   
 k)  $(\ln \sqrt{x} + x^5) \cdot \sec^2(e^{3x+1}) \cdot e^{3x+1} \cdot 3 + \tan(e^{3x+1}) \cdot \left(\frac{1}{2x} + 5x^4\right)$   
 l)  $\frac{3(x^2+3)\sin^2 x \cos x \ln(x^2+3) - 2x \sin^3 x}{(x^2+3)(\ln(x^2+3))^2}$   
 m)  $\frac{2x \tan^{-1} x - 1}{(\tan^{-1} x)^2}$   
 n)  $\frac{1}{3\sqrt{(x^2 \sin^{-1} x)^2}} \left(\frac{x^2}{\sqrt{1-x^2}} + 2x \sin^{-1} x\right)$   
 o)  $\sinh(\sinh(\tanh x)) \cdot \cosh(\tanh x) \cdot \operatorname{sech}^2 x$

17.

- a)  $y + 2 = -2(x - 0)$  or  $y = -2x - 2$
- b)  $y - 1 = 2\left(x - \frac{\pi}{4}\right)$  or  $y = 2x - \frac{\pi}{2} + 1$
- c)  $y = 2x$

18.

- a)  $y'' = 4e^x + 12x + \frac{2}{x^3}$
- b)  $y'' = -x^2 \cos x - 4x \sin x + 2 \cos x$