_____ / 15 total points

Quiz #3

Name: ____________________

Math 180, Section 5, Prof. Beydler

Thursday, November 9, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (3 points) Find the absolute maximum and minimum values of $f(x) = x \ln x$ on the interval $[0.1, 2]$. Round your answers to 2 decimal places.

$f'(x) = x \cdot \frac{1}{x} + 1 \cdot \ln x = 1 + \ln x$

Absolute maximum value: 1.38

Absolute minimum value: −0.37

$\underline{f'=0:}$

$1 + \ln x = 0$

$e^{\ln x} = e^{-1}$

$x = \frac{1}{e}$

$\underline{f' \text{ DNE:}}$

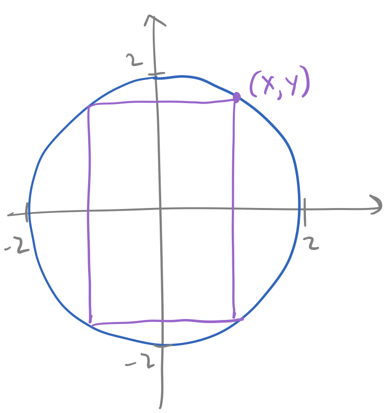
Nowhere (in domain of f)

x	$f(x)$
$\frac{1}{e}$	$-\frac{1}{e} \approx -0.37$
0.1	-0.23
2	1.38

$\frac{1}{e} \ln\left(\frac{1}{e}\right) = -\frac{1}{e}$

2. (5 points) Find the largest possible volume of a cylinder that's inscribed in a sphere of radius 2.

Largest volume: $\frac{32\pi}{3\sqrt{3}}$



$V(x, y) = \pi x^2 \cdot (2y)$

$x^2 + y^2 = 4$

$x^2 = 4 - y^2$

$V(y) = 2\pi(4 - y^2)y$

$= 8\pi y - 2\pi y^3$

$V'(y) = 8\pi - 6\pi y^2$

$\underline{V' = 0:}$

$8\cancel{\pi} - 6\cancel{\pi} y^2 = 0$

$y^2 = \frac{4}{3}$

$y = \frac{2}{\sqrt{3}}, \cancel{y = -\frac{2}{\sqrt{3}}}$

$V\left(\frac{2}{\sqrt{3}}\right) = 2\pi\left(4 - \left(\frac{2}{\sqrt{3}}\right)^2\right)\frac{2}{\sqrt{3}}$

$= \frac{4\pi}{\sqrt{3}}\left(\frac{8}{3}\right)$

3. (3 points) Find the most general antiderivative for $f(x) = 5\sec^2 x + \sin 3x + \frac{2}{x^2} - \frac{4}{\sqrt[3]{x}} + 2^x$.

Answer: $5\tan x - \frac{1}{3}\cos 3x - \frac{2}{x} - 6x^{2/3} + \frac{2^x}{\ln 2} + C$

$$f(x) = 5\sec^2 x + \sin 3x + 2x^{-2} - 4x^{-1/3} + 2^x$$

$$F(x) = 5\tan x - \frac{1}{3}\cos 3x + 2 \cdot \frac{x^{-1}}{-1} - 4 \cdot \frac{x^{2/3}}{(2/3)} + \frac{2^x}{\ln 2} + C$$

4. (4 points) $\int 3x e^{x^2+2}\, dx$

$u = x^2 + 2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$\frac{3}{2} du = 3x\, dx$

$= \int e^u \cdot \frac{3}{2} du$

$= \frac{3}{2} e^u + C$

Answer: $\frac{3}{2} e^{x^2+2} + C$