_____ / 15 total points

Quiz #2

Name: _____

Math 180, Section 5, Prof. Beydler

Thursday, October 12, 2017

Directions: Show all work. No books or notes. A scientific calculator is allowed. Your desk and lap must be clear (no phones, notebooks, etc.). Write your answers in the indicated places, or box your answers. Good luck!

1. (4 points) Find an equation of the tangent line to $\sin(xy) = y + 1$ at $(0, 1)$.

Answer: $y = x + 1$ (or $y - 1 = x$)

$$\frac{d}{dx}(\sin(xy)) = \frac{d}{dx}(y) + \frac{d}{dx}(1)$$

$$\cos(xy) \cdot \left(x\frac{dy}{dx} + 1 \cdot y\right) = \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} - \frac{dy}{dx} = -y\cos(xy)$$

$$\frac{dy}{dx}(x\cos(xy) - 1) = -y\cos(xy)$$

$$\frac{dy}{dx} = \frac{-y\cos(xy)}{x\cos(xy) - 1}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-1 \cdot \cos(0 \cdot 1)}{0 \cdot \cos(0 \cdot 1) - 1} = 1$$

$$y - 1 = 1(x - 0)$$

$$y = x + 1$$

2. (3 points) Use logarithmic differentiation to find the derivative of y with respect to x.

$$y = \frac{e^{x-1}}{\sqrt{x}(2x+3)^3}$$

$\frac{dy}{dx} =$ $y\left(1 - \frac{1}{2x} - \frac{6}{2x+3}\right)$

$$\ln y = \ln \frac{e^{x-1}}{x^{1/2}(2x+3)^3}$$

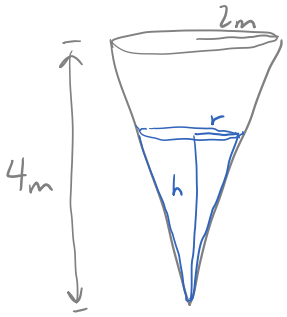
$$= \ln e^{x-1} - \frac{1}{2}\ln x - 3\ln(2x+3)$$

$$= x - 1 - \frac{1}{2}\ln x - 3\ln(2x+3)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 - \frac{1}{2} \cdot \frac{1}{x} - 3 \cdot \frac{1}{2x+3} \cdot 2$$

$$\frac{dy}{dx} = y\left(1 - \frac{1}{2x} - \frac{6}{2x+3}\right)$$

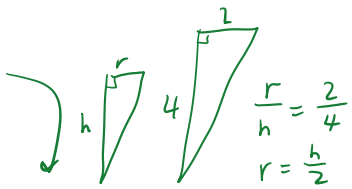
3. (4 points) A conical water tank that stands point down has a base radius of 2 m and a height of 4 m. If water is pumped in at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep. Be sure to write the units for your answer.



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{\pi}{12}h^3$



$\frac{r}{h} = \frac{2}{4}$

$r = \frac{h}{2}$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{\pi(3)^2}\cdot(2) = \frac{8}{9\pi}$

Answer: $\frac{8}{9\pi}$ m/min

4. The position of a particle is given by the equation $s(t) = t^3 - 9t^2 + 24t + 1$ (where $t \geq 0$ is measured in seconds and s is measured in meters).

a) (1 point) When is the particle at rest?

$v(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 3(t-2)(t-4)$

$t = 2$ $\quad$ $t = 4$

Answer: $t = 2, 4$ sec

b) (1 point) Find the total distance traveled during the first 6 seconds. Be sure to include the units for your answer.

$|s(2) - s(0)| = |21 - 1| = 20$

$|s(4) - s(2)| = |17 - 21| = 4$

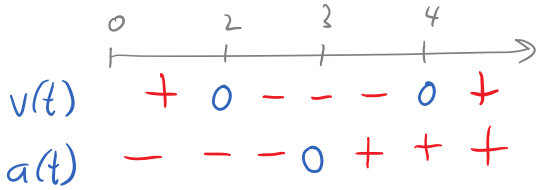
$|s(6) - s(4)| = |37 - 17| = 20$

} Add

Answer: 44 m

c) (2 points) When is the particle speeding up? When is it slowing down?

$a(t) = 6t - 18 = 6(t-3)$



Speeding up: $2 < t < 3,\ t > 4$

Slowing down: $0 \leq t < 2,\ 3 < t < 4$