Test #2 Review Exercise Answers

1.

a) 0

b) $\sqrt{\frac{2}{3}}$

c) undefined

d) $\frac{\sqrt{x}}{\sqrt{x-1}}$; Domain: $[0,1) \cup (1,\infty)$

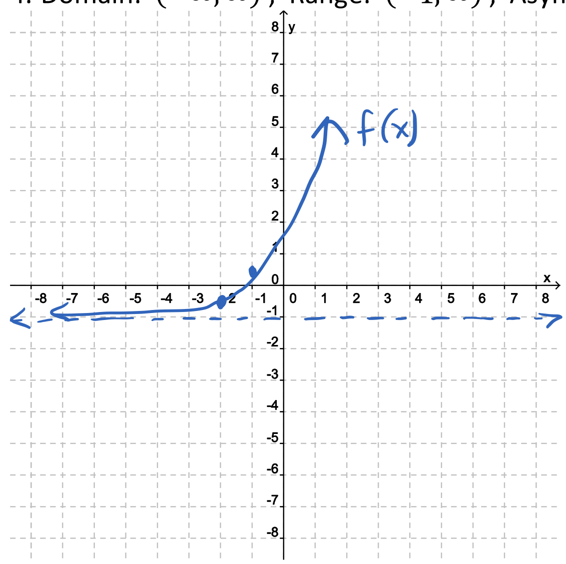
e) $\sqrt{\frac{x}{x-1}}$; Domain: $(-\infty,0] \cup (1,\infty)$

f) $\sqrt[4]{x}$; Domain: $[0,\infty)$

2. $f^{-1}(x) = \frac{2-4x}{3x+1}$; Domain: $\left(-\infty,-\frac{1}{3}\right) \cup \left(-\frac{1}{3},\infty\right)$

3. $f^{-1}(x) = (x-1)^2 - 5$; Domain: $(-\infty,1]$ (By the way, $(1-x)^2 = (x-1)^2$.)

4. Domain: $(-\infty,\infty)$; Range: $(-1,\infty)$; Asymptote: $y = -1$



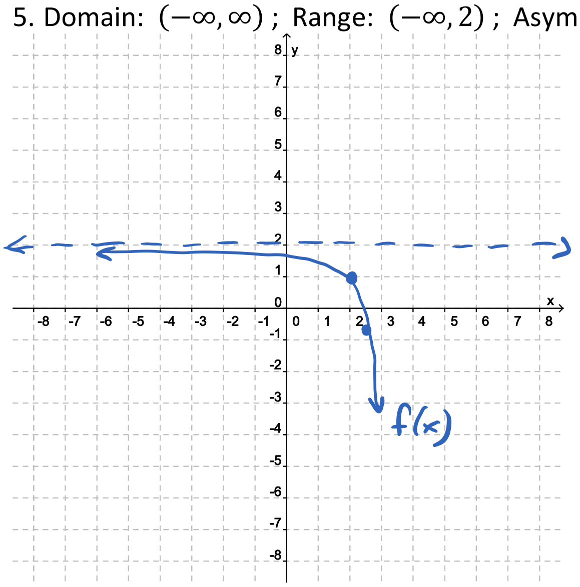
3^x

3^{x+2} Shift left 2

$\frac{1}{2}\cdot 3^{x+2}$ Shrink vertically by factor of $\frac{1}{2}$

$\frac{1}{2}\cdot 3^{x+2} - 1$ Shift down 1

5. Domain: $(-\infty,\infty)$; Range: $(-\infty,2)$; Asymptote: $y = 2$



e^x

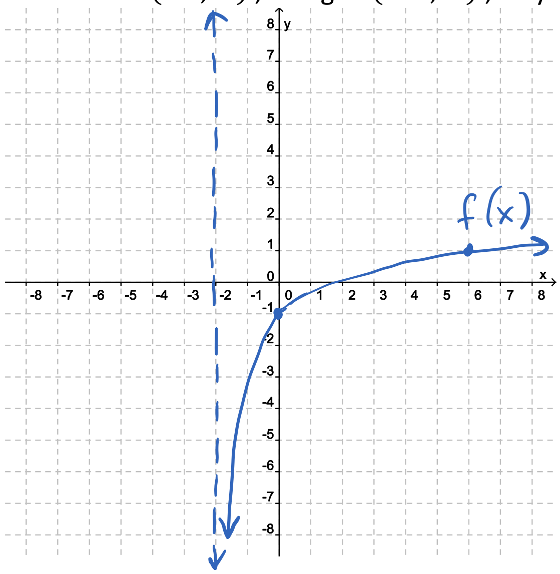
e^{x-4} Shift right 4

e^{2x-4} Shrink horizontally by factor of $\frac{1}{2}$

$-e^{2x-4}$ Reflect about x-axis

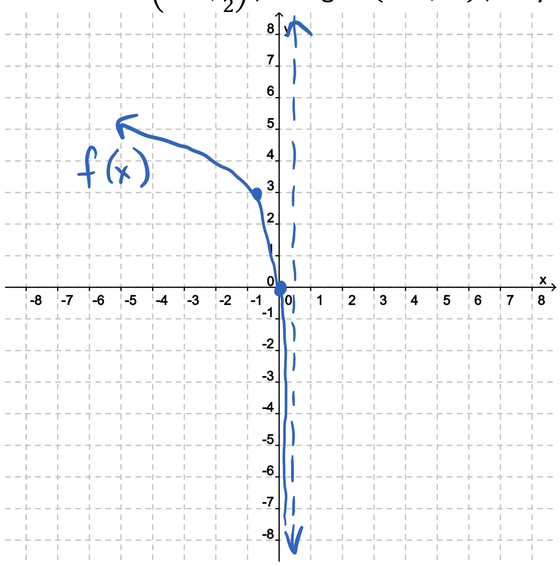
$2 - e^{2x-4}$ Shift up 2

6. Domain: $(-2, \infty)$; Range: $(-\infty, \infty)$; Asymptote: $x = -2$



$\log_4 x$
$\log_4 (x+1)$ Shift left 1
$\log_4 \left(\frac{1}{2}x+1\right)$ Stretch horizontally by factor of 2
$2\log_4 \left(\frac{1}{2}x+1\right)$ Stretch vertically by factor of 2
$2\log_4 \left(\frac{1}{2}x+1\right) - 1$ Shift down 1

7. Domain: $\left(-\infty, \frac{1}{2}\right)$; Range: $(-\infty, \infty)$; Asymptote: $x = \frac{1}{2}$



$\ln x$
$\ln (x+1)$ Shift left 1
$\ln (2x+1)$ Shrink horizontally by factor of $\frac{1}{2}$
$\ln (-2x+1)$ Reflect about y-axis
$3\ln (-2x+1)$ Stretch vertically by factor of 3

8. $(2, \infty)$

9. $(1,2) \cup (2, \infty)$

10. $\ln(2x+1) + \frac{1}{2}\ln(x+2) - \ln 5 + 2 - 3\ln x - 4\ln(3-x)$

11. $\frac{1}{3}\left(x + 3\log_2(x-5) - 3 - \frac{1}{2}\log_2 x - 5\log_2(3x+7)\right)$

12. $\log_4 \frac{\sqrt[3]{x-3}}{\sqrt{x}(x+1)}$

13. $\ln \frac{(x+5)^3}{x(2x-1)}$

14. $x = \frac{2\ln 3 + 3\ln 5}{2\ln 5 - \ln 3}$

15. 0, 2

16. $\frac{\ln\frac{1}{2}}{\ln 3}$ (or $\log_3 \frac{1}{2}$)

17. $5 \pm \sqrt{7}$

18. $\frac{16}{15}$

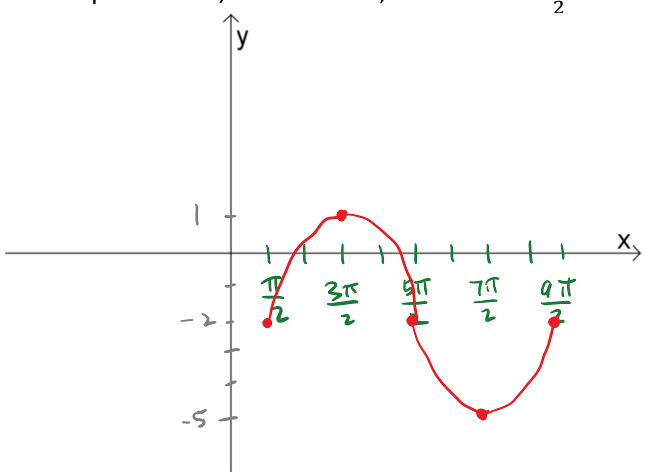
19. 2

20. The population will be one million in the year 2385. (Note: $k \approx 0.027465$)

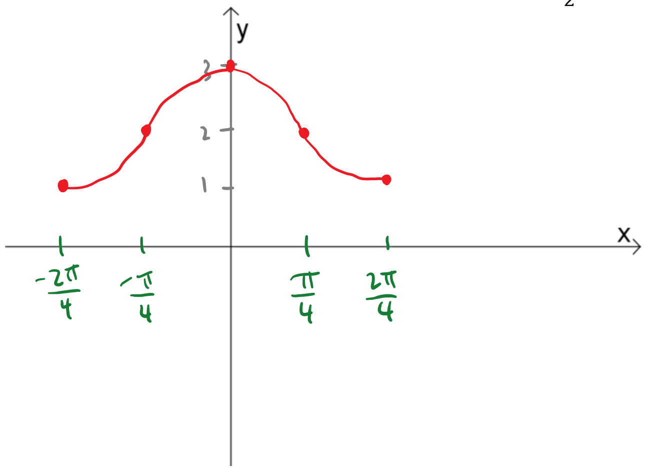
21. Model: $T(t) = 70 + 255e^{-0.033686t}$; The yam will be $150°F$ approximately 34.413 minutes after taking it out of the oven.

22.
  a) $-\frac{\sqrt{3}}{2}$
  b) $-\frac{\sqrt{2}}{2}$
  c) undefined
  d) 2
  e) $-\frac{2}{\sqrt{3}}$
  f) $-\frac{1}{\sqrt{3}}$
  g) 0
  h) 0

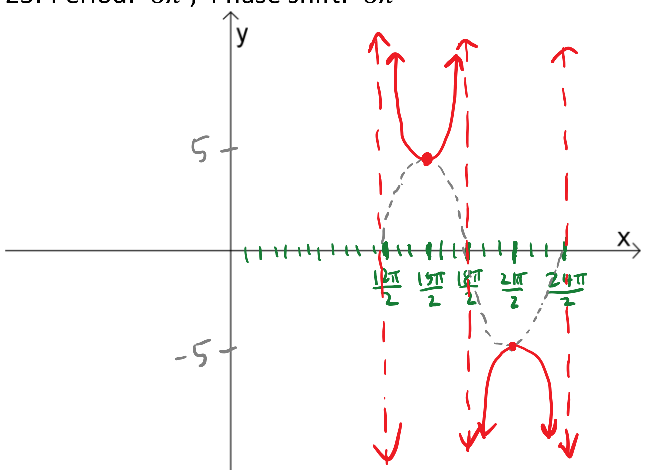
23. Amplitude: 3 ; Period: 4π ; Phase shift: $\frac{\pi}{2}$



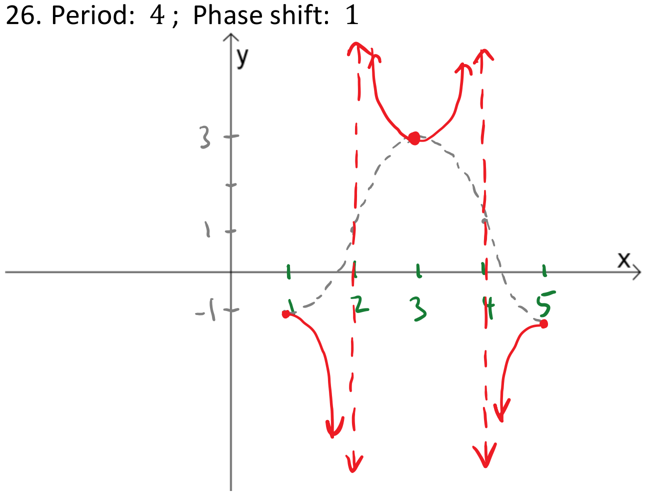
24. Amplitude: 1 ; Period: π ; Phase shift: $-\frac{\pi}{2}$



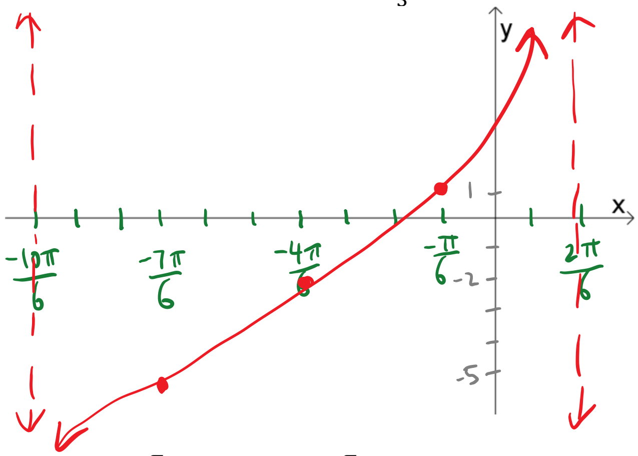
25. Period: 6π ; Phase shift: 6π



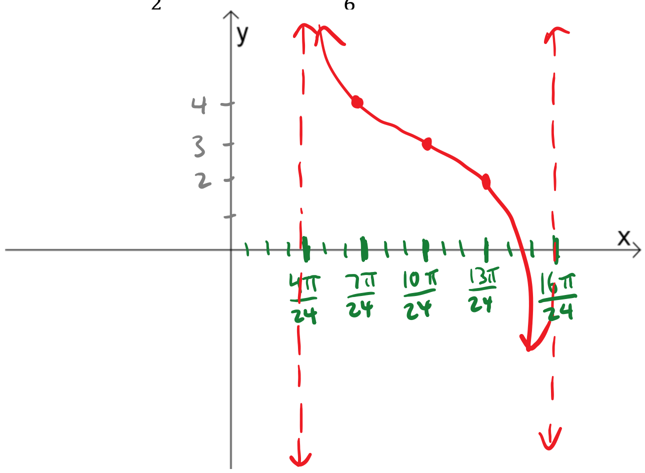
26. Period: 4 ; Phase shift: 1



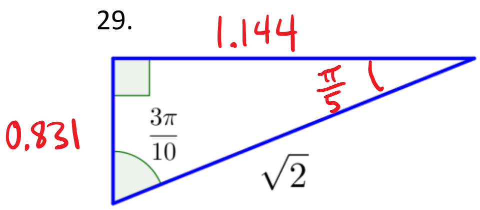
27. Period: 2π ; Phase shift: $-\frac{2\pi}{3}$



28. Period: $\frac{\pi}{2}$; Phase shift: $\frac{\pi}{6}$



29.



30. $-\frac{1}{\sqrt{6}}$

31. $\frac{1}{2\sqrt{2}}$

32.

a) $\frac{\pi}{3}$
b) $\frac{3\pi}{4}$
c) $-\frac{\pi}{3}$
d) undefined
e) $\frac{2\pi}{3}$
f) π
g) 0

h) $\frac{\pi}{3}$
i) $\frac{3\pi}{4}$
j) $\frac{2\pi}{3}$
k) $\frac{\pi}{6}$

33.
a) $-\frac{\sqrt{5}}{2}$
b) $\frac{\sqrt{5}}{2}$
c) $-\frac{2}{\sqrt{3}}$

34.
a) $\frac{\sqrt{1-x^2}}{x}$
b) $\frac{1}{\sqrt{1-x^2}}$

35. $RHS = \sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1+\sin x}{\cos x} = \frac{(1+\sin x)\cdot(1-\sin x)}{\cos x\cdot(1-\sin x)} = \frac{1-\sin^2 x}{\cos x\cdot(1-\sin x)} = \frac{\cos^2 x}{\cos x\cdot(1-\sin x)} = \frac{\cos x}{1-\sin x} = LHS$

36. $LHS = \frac{1+\tan x}{1-\tan x} = \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} = \frac{\left(1+\frac{\sin x}{\cos x}\right)\cdot\cos x}{\left(1-\frac{\sin x}{\cos x}\right)\cdot\cos x} = \frac{\cos x+\sin x}{\cos x-\sin x} = RHS$

37. $LHS = \frac{1+\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{1+\cos\alpha} = \frac{(1+\cos\alpha)^2+\sin^2\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{1+2\cos\alpha+\cos^2\alpha+\sin^2\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{1+2\cos\alpha+1}{\sin\alpha(1+\cos\alpha)} = \frac{2+2\cos\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{2(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)} = \frac{2}{\sin\alpha} = 2\csc\alpha = RHS$

38. $LHS = \frac{1}{\csc\theta+\cot\theta} = \frac{1}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}} = \frac{1\cdot\sin\theta}{\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)\cdot\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

$RHS = \csc\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1-\cos\theta}{\sin\theta} = \frac{(1-\cos\theta)\cdot(1+\cos\theta)}{\sin\theta\cdot(1+\cos\theta)} = \frac{1-\cos^2\theta}{\sin\theta\cdot(1+\cos\theta)} = \frac{\sin^2\theta}{\sin\theta\cdot(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$

Thus, since $LHS = \frac{\sin\theta}{1+\cos\theta} = RHS$, we have proven the identity.

39. $LHS = \tan^2 x - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x\cos^2 x}{\cos^2 x} = \frac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x} = \frac{\sin^2 x\left(1-\cos^2 x\right)}{\cos^2 x} = \frac{\sin^2 x\sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x}\cdot\sin^2 x = \tan^2 x\sin^2 x = RHS$

40. $LHS = \frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y+\sin x\sin y}$

$RHS = \frac{\tan x+\tan y}{1+\tan x\tan y} = \frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{1+\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}} = \frac{\left(\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}\right)\cdot\cos x\cos y}{\left(1+\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}\right)\cdot\cos x\cos y} = \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y+\sin x\sin y}$

Thus, since $LHS = \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y+\sin x\sin y} = RHS$, we have proven the identity.

41. $LHS = 2\sin^2 2x + \cos 4x = 2\sin^2 2x + 1 - 2\sin^2 2x = 1 = RHS$

42. $\frac{24}{25}$

43. $-\frac{5}{13}$

44. $\frac{xy+\sqrt{1-y^2}}{\sqrt{x^2+1}}$

45. $\frac{2x^2-1}{2x\sqrt{1-x^2}}$

46. $\cos 2x = -\frac{15}{17},\ \sin 2x = \frac{8}{17},\ \sin\frac{x}{2} = \sqrt{\frac{\sqrt{17}+1}{2\sqrt{17}}}$

47. $\sin 2x = -\frac{4\sqrt{2}}{9},\ \tan 2x = \frac{4\sqrt{2}}{7},\ \cos\frac{x}{2} = -\frac{2}{\sqrt{6}}$

48. $\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$