Test #2 (Part 2, Calculator Okay)

Name: _____

Math 160, Prof. Beydler

Thursday, October 25, 2018

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. Let $f(x) = \frac{x}{x-1}$ and $g(x) = \sqrt{x}$.

   a) (1 point) Find $(f \circ f)(2)$.

$f(f(2)) = f\left(\frac{2}{2-1}\right) = f(2) = \frac{2}{2-1} = 2$

Answer: 2

   b) (3 points) Find $f \circ g$ and its domain.

$f(g(x)) = f(\sqrt{x}) = \frac{\sqrt{x}}{\sqrt{x}-1}$

$(f \circ g)(x) =$ $\frac{\sqrt{x}}{\sqrt{x}-1}$

Domain of $f \circ g$: $[0, 1) \cup (1, \infty)$

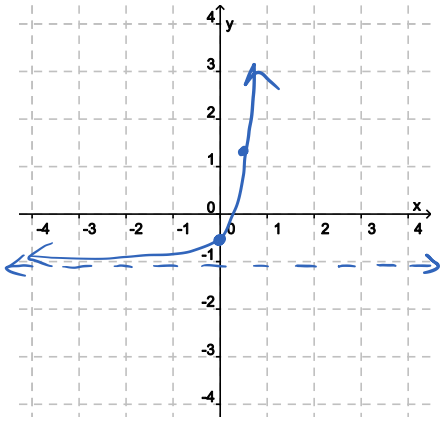
(or $\{x \mid x \geq 0 \text{ and } x \neq 1\}$)

Need $x \geq 0$ and $\sqrt{x} - 1 \neq 0$

$\sqrt{x} \neq 1$

$x \neq 1$

2. (5 points) Graph $f(x) = \frac{1}{2}e^{2x} - 1$. State the domain, range, and asymptote. Be sure to describe the transformations to the basic function.



e^x

e^{2x} Shrink horiz by factor of $\frac{1}{2}$

$\frac{1}{2}e^{2x}$ Shrink vert by factor of $\frac{1}{2}$

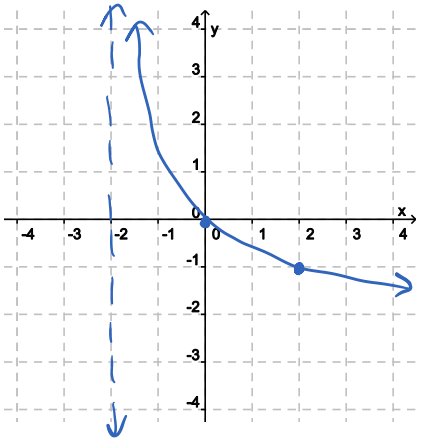
$\frac{1}{2}e^{2x} - 1$ Shift down 1

Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Asymptote: $y = -1$

3. (5 points) Graph $f(x) = -\log_2\left(\frac{x}{2}+1\right)$. State the domain, range, and asymptote. Be sure to describe the transformations to the basic function.



$\log_2 x$

$\log_2(x+1)$ Shift left 1

$\log_2\left(\frac{x}{2}+1\right)$ Stretch horiz by factor of 2

$-\log_2\left(\frac{x}{2}+1\right)$ Reflect about x-axis

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -2$

4. (3 points) Solve: $5^{2x} - 2 \cdot 5^x - 3 = 0$

$(5^x+1)(5^x-3) = 0$

$5^x = -1$ ~~$= -1$~~ No solutions here

$5^x = 3$

$\ln 5^x = \ln 3$

$x \ln 5 = \ln 3$

$x = \frac{\ln 3}{\ln 5}$

Let $u = 5^x$

$u^2 - 2u - 3 = 0$

$(u+1)(u-3) = 0$

Answer: $\frac{\ln 3}{\ln 5}$ (or $\log_5 3$)

5. (3 points) Solve: $\log_2(2x+3) = 3 + \log_2(x-1)$

$\log_2(2x+3) - \log_2(x-1) = 3$

$2^{\log_2\left(\frac{2x+3}{x-1}\right)} = 2^3$

$\frac{2x+3}{x-1} = 8$

$2x+3 = 8x-8$

$11 = 6x$

$x = \frac{11}{6}$

Answer: $\frac{11}{6}$

6. (5 points) A cast iron skillet is $500°F$ when you take it out of the oven. Ten minutes later, the skillet is $400°F$. Suppose the room temperature is $70°F$. First, use Newton's Law of Cooling ($T(t) = T_s + (T_0 - T_s)e^{-kt}$) to find a function that models the temperature of the skillet t minutes after your initial temperature reading. Then, use the function to predict when the skillet will be $200°F$ (round your answer to the nearest minute). While solving, round k to 5 significant figures.

Model $T(t) =$ $70 + 430e^{-0.026469t}$

When skillet will be $200°F$: 45 min

$T(t) = 70 + (500 - 70)e^{-kt}$

$= 70 + 430e^{-kt}$

$T(t) = 70 + 430e^{-0.026469t}$

$T(10) = 400$

$70 + 430e^{-k(10)} = 400$

$430e^{-10k} = 330$

$\ln e^{-10k} = \ln \frac{33}{43}$

$-10k = \ln \frac{33}{43}$

$k = \frac{\ln 33/43}{-10} \approx 0.026469$

When will $T(t) = 200$?

$70 + 430e^{-0.026469t} = 200$

$430e^{-0.026469t} = 130$

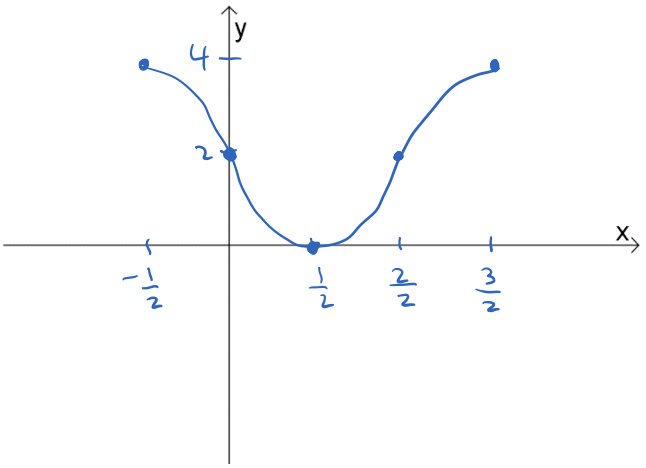
$\ln e^{-0.026469t} = \ln \frac{13}{43}$

$-0.026469t = \ln \frac{13}{43}$

$t = \frac{\ln \frac{13}{43}}{-0.026469} \approx 45$

7. (4 points) Find the amplitude, period, and phase shift of $y = 2 + 2\cos\left(\pi x + \frac{\pi}{2}\right)$ and graph one complete period. Be sure to find the 5 key points.

$y = 2 + 2\cos\pi\left(x + \frac{1}{2}\right)$



Amplitude: 2

Period: 2

Phase shift: $-\frac{1}{2}$

QP: $\frac{2}{4} = \frac{1}{2}$

x-values: $-\frac{1}{2}$ 0 $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$

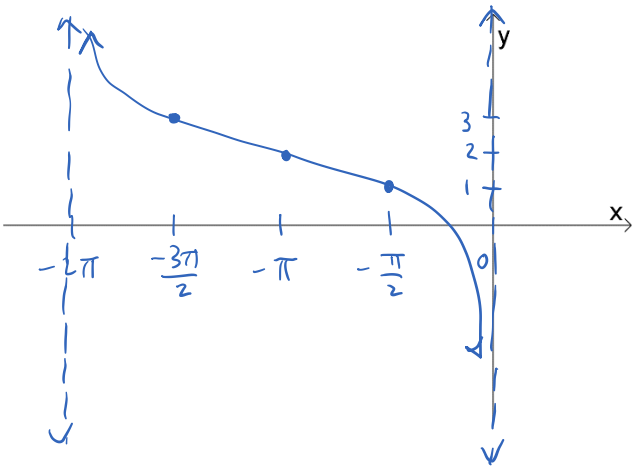
y-values: 1 0 −1 0 1 ↘ ·2

2 0 −2 0 2 ↘ +2

4 2 0 2 4

8. (4 points) Find the period and phase shift of $y = \cot\left(\frac{1}{2}x + \pi\right) + 2$ and graph one complete period. Be sure to find the 5 key points/asymptotes.

$y = \cot\frac{1}{2}(x + 2\pi) + 2$



$\frac{\pi}{(\frac{1}{2})}$

Period: 2π

Phase shift: -2π

QP: $\frac{2\pi}{4} = \frac{\pi}{2}$

x-values: -2π $-\frac{3\pi}{2}$ $-\pi$ $-\frac{\pi}{2}$ 0

y-values: 1 0 −1 ↘ +2

3 2 1

Note: Be sure to double-check your work. And don't forget to turn in your homework! ☺