_____ / 70 total points (both parts)

Test #2 (Part 1, No Calculator) Name: ______________________________

Math 160, Prof. Beydler Thursday, October 25, 2018

Directions: Show all work. No calculator, books, or notes. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. When you're finished with Part 1, please turn it in, take a bathroom break, get your calculator out, and start Part 2. Good luck!

1. (3 points) Find the inverse of $f(x) = \frac{2x-1}{3-x}$. Be sure to state the domain of $f^{-1}(x)$.

$$y = \frac{2x-1}{3-x}$$

$$x = \frac{2y-1}{3-y}$$

$$3x - xy = 2y - 1$$

$$3x + 1 = 2y + xy$$

$$3x + 1 = y(2+x)$$

$$y = \frac{3x+1}{2+x}$$

$f^{-1}(x) =$ $\frac{3x+1}{x+2}$

Domain of $f^{-1}(x)$: $(-\infty, -2) \cup (-2, \infty)$

(or $\{x \mid x \neq -2\}$)

2. (3 points) Find the domain of $f(x) = \log_3(x+3) + \sqrt{\frac{x}{x+2}}$

Need $x + 3 > 0$ and $\frac{x}{x+2} \geq 0$

$x > -3$

Domain: $(-3, -2) \cup [0, \infty)$

-2 0

$\frac{x}{x+2}$: $+$ Und $-$ 0 $+$

3. (3 points) Expand using the properties of logarithms: $\ln \sqrt{\frac{e^x}{x^3(3x+2)^4}}$

Answer: $\frac{1}{2}x - \frac{3}{2}\ln x - 2\ln(3x+2)$

$$\frac{1}{2}\ln \frac{e^x}{x^3(3x+2)^4} = \frac{1}{2}\left[\ln e^x - \ln x^3 - \ln(3x+2)^4\right]$$

4. (3 points) Write as a single logarithm and simplify:

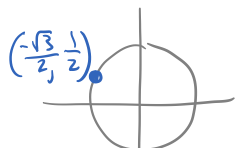
$\frac{1}{3}\log(x+2) - 2\log(2x-3) + \log(x+1) - \frac{1}{4}\log x$

$= \log(x+2)^{1/3} - \log(2x-3)^2 + \log(x+1) - \log x^{1/4}$

Answer: $\log \frac{\sqrt[3]{x+2}\,(x+1)}{(2x-3)^2\,\sqrt[4]{x}}$

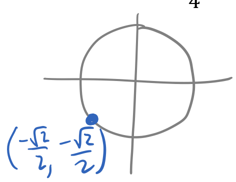
5. Find the exact value <u>without a calculator</u>.

a) (1 point) $\sin\left(-\frac{7\pi}{6}\right)$



Answer: $\frac{1}{2}$

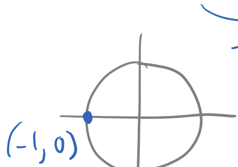
b) (1 point) $\cos\frac{5\pi}{4}$



Answer: $-\frac{\sqrt{2}}{2}$

c) (1 point) $\cot 41\pi = \cot\pi = \frac{-1}{0}$

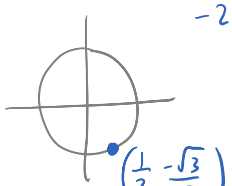
-40π



Answer: undefined

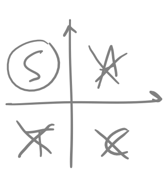
d) (1 point) $\csc\frac{11\pi}{3} = \csc\frac{5\pi}{3}$

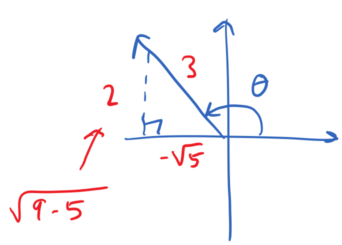
-2π



Answer: $-\frac{2}{\sqrt{3}}$ $\left(\text{or } -\frac{2\sqrt{3}}{3}\right)$

6. (2 points) Suppose $\cos\theta = -\frac{\sqrt{5}}{3}$ and $\cot\theta < 0$. Find $\csc\theta$.





Answer: $\frac{3}{2}$

7. Find the exact value in radians <u>without a calculator</u>.

a) (1 point) $\cos^{-1}\frac{1}{\sqrt{2}}$

Answer: $\frac{\pi}{4}$

b) (1 point) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Answer: $-\frac{\pi}{6}$

c) (1 point) $\sin^{-1}\pi$

Answer: undefined

d) (1 point) $\sin^{-1}\left(\sin\frac{11\pi}{6}\right)$

$= \sin^{-1}\left(\frac{-1}{2}\right)$

Answer: $\frac{-\pi}{6}$

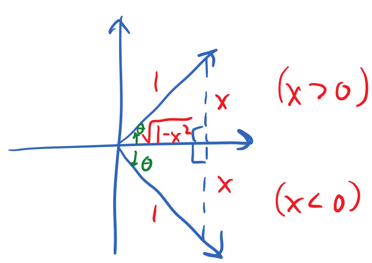
8. (2 points) Rewrite each expression as an algebraic expression in x.

$\cot(\underbrace{\sin^{-1}x}_{\theta}) = \cot\theta$

Answer: $\frac{\sqrt{1-x^2}}{x}$

$\sin^{-1}x = \theta \leftarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\theta = x = \frac{x}{1}$



9. (3 points) Find the exact value of $\cos\left(\underbrace{2\tan^{-1}\frac{5}{2}}_{\theta}\right)$ <u>without a calculator</u>.

$= \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

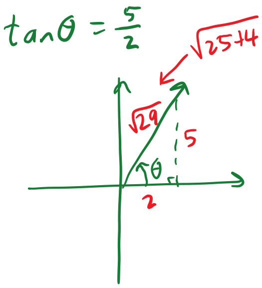
$= \left(\frac{2}{\sqrt{29}}\right)^2 - \left(\frac{5}{\sqrt{29}}\right)^2$

$= \frac{4}{29} - \frac{25}{29}$

$= \frac{-21}{29}$

$\tan^{-1}\frac{5}{2} = \theta$

$\tan\theta = \frac{5}{2}$



Answer: $\frac{-21}{29}$

10. (3 points) Prove the identity: $\frac{1}{\tan\theta+\cot\theta} = \sin\theta\cos\theta$

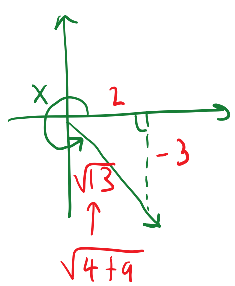
$$LHS = \frac{1}{\tan\theta + \cot\theta}$$

$$= \frac{(1)\cdot \sin\theta\cos\theta}{\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\cdot \sin\theta\cos\theta}$$

$$= \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \frac{\sin\theta\cos\theta}{1} = \sin\theta\cos\theta = RHS \quad \blacksquare$$

11. (4 points) Suppose $\cot x = -\frac{2}{3}$ and $270° < x < 360°$. Find $\sin 2x$ and $\cos\frac{x}{2}$.



$$\sin 2x = 2\sin x\cos x$$

$$= 2\left(\frac{-3}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right)$$

$$= \frac{-12}{13}$$

$\sin 2x =$ $\frac{-12}{13}$

$\cos\frac{x}{2} =$ $-\sqrt{\frac{\sqrt{13}+2}{2\sqrt{13}}}$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{\left(1+\left(\frac{2}{\sqrt{13}}\right)\right)\cdot\sqrt{13}}{(2)\cdot\sqrt{13}}} = -\sqrt{\frac{\sqrt{13}+2}{2\sqrt{13}}}$$

$270° < x < 360°$

$135° < \frac{x}{2} < 180°$

So, $\cos\frac{x}{2} < 0$

12. (3 points) Express $\cos^4 x$ in terms of the first power of cosine.

Answer: $\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

$$\cos^4 x = (\cos^2 x)^2$$

$$= \left(\frac{1+\cos 2x}{2}\right)^2$$

$$= \frac{1}{4}\left(1 + 2\cos 2x + \cos^2 2x\right)$$

$$= \frac{1}{4}\left(1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right)$$

$$= \frac{1}{4}\left(1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right)$$

$$= \frac{1}{4}\left(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right)$$