

# Test #1 (Part 2, Calculator Okay)

Math 160, Prof. Beydler

Name: \_\_\_\_\_

Thursday, September 27, 2018

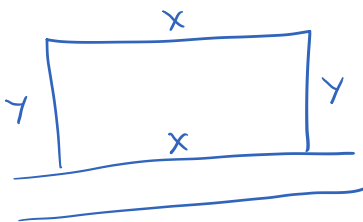
**Directions:** Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, no smart watches, etc.). If you have a phone in your lap or on your chair, it is considered cheating, and you will receive a zero on this test. Write your answers in the indicated places, or box your answers. Good luck!

1. (2 points) Find the average rate of change of  $f(x) = 3x^2 - x$  between  $x = -1$  and  $x = 2$ .

$$\begin{aligned}\frac{f(2) - f(-1)}{2 - (-1)} &= \frac{3(2)^2 - 2 - (3(-1)^2 - (-1))}{3} \\ &= \frac{10 - 4}{3}\end{aligned}$$

Answer: 2

2. (2 points) You buy a 1000-square-foot, rectangular piece of property with one side against a straight road. You'd like to protect your property with fencing, most of which costs \$6 per foot. However, the fencing next to the road must be sturdier and costs \$10 per foot. Find a function in one variable that models the cost of fencing your property.



Answer:  $C(x) = 16x + \frac{12000}{x}$

$$C(x, y) = 6(x + y + y) + 10x$$

$$C(x, y) = 16x + 12y$$

$$C(x) = 16x + 12\left(\frac{1000}{x}\right)$$

$$C(x) = 16x + \frac{12000}{x}$$

$$\begin{aligned}xy &= 1000 \\ y &= \frac{1000}{x}\end{aligned}$$

3. Let  $f(x) = -x^2 - 4x + 3$ .

a) (2 points) Express  $f$  in standard form.

$$f(x) = -(x^2 + 4x + 4) + 3 + 4$$

$$= -(x+2)^2 + 7$$

Answer:  $f(x) = \underline{-(x+2)^2 + 7}$

b) (1 point) Find the vertex of  $f$ . Vertex:  $(-2, 7)$

c) (2 points) Sketch the graph of  $f$  to the right. Be sure to plot the vertex and  $y$ -intercept.

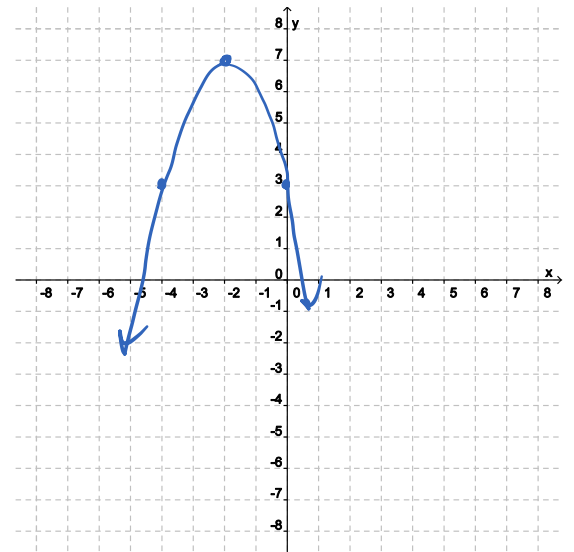
d) (1 point) Find the maximum or minimum value of  $f$ .

Circle one: maximum minimum

The value is: 7

e) (1 point) Find the range of  $f$ .

Range:  $(-\infty, 7]$



4. (2 points) A frog jumps up. The height of the frog  $t$  seconds after jumping is given by the function  $y(t) = 2.45t - 4.9t^2$  (in meters). What is maximum height attained by the frog? When does this happen?

$$t = \frac{-2.45}{2(-4.9)} = 0.25$$

$$y(0.25) = 2.45(0.25) - 4.9(0.25)^2$$

$$= 0.30625$$

Maximum height: 0.30625 m

Time of maximum height:  $t = \underline{0.25 \text{ sec.}}$

5. Let  $f(x) = -2(x - 3)^2(x + 1)$ .

a) (1 point) Find the  $x$ -intercepts of  $f$ .  $x$ -intercepts: 3, -1

b) (1 point) Find the  $y$ -intercept of  $f$ .  $y$ -intercept: -18

$$f(0) = -2(-3)^2(1) = -18$$

c) (1.5 points) Find a test point between the  $x$ -intercepts of  $f$ , as well as a test point before the first  $x$ -intercept, and a test point after the last  $x$ -intercept.

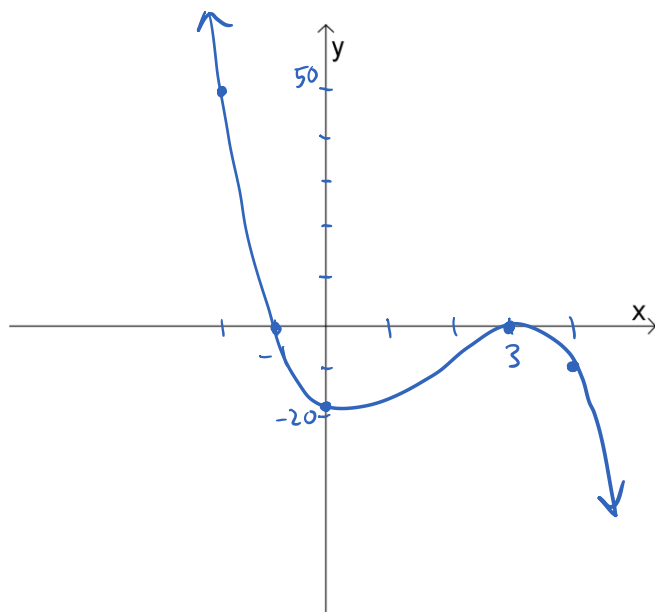
$x$	$f(x)$
-2	50
0	-18
4	-10

d) (1 point) Determine the end behavior of  $f$ .

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$   $-\infty$

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$   $\infty$

e) (2 points) Sketch the graph of  $f$ .



6. (4 points) Find the complete factorization and all zeros of  $P(x) = 2x^4 - 11x^3 + 15x^2 - 7x + 1$ . If any zeros have a multiplicity of 2 or more, state so.

Complete factorization:  $P(x) = \underline{(x-1)(2x-1)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))}$

Zeros:  $1, \frac{1}{2}, 2 \pm \sqrt{3}$

Possible rational zeros:  $\pm 1, \pm \frac{1}{2}$

$$\begin{array}{r|rrrrr} 1 & 2 & -11 & 15 & -7 & 1 \\ & & 2 & -9 & 6 & -1 \\ \hline & 2 & -9 & 6 & -1 & 0 \end{array}$$

$$\begin{aligned} P(x) &= (x-1)(2x^3 - 9x^2 + 6x - 1) \\ &= (x-1)\left(x - \frac{1}{2}\right)\left(2x^2 - 8x + 2\right) \\ &= (x-1)(2x-1)(x^2 - 4x + 1) \end{aligned}$$

$$\frac{1}{2} \left| \begin{array}{r|rrrr} 2 & -9 & 6 & -1 \\ & 1 & -4 & 1 \\ \hline 2 & -8 & 2 & 0 \end{array} \right.$$

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{4 \pm \sqrt{12}}{2} \\ &= \frac{4 \pm 2\sqrt{3}}{2} \\ &= 2 \pm \sqrt{3} \end{aligned}$$

7. Let  $f(x) = \frac{x+1}{2x^2+3x-2}$ .

a) (1 point) Find the domain of  $f$ .

$$\frac{x+1}{(2x-1)(x+2)}$$

Domain:  $\{x \mid x \neq \frac{1}{2}, x \neq -2\}$   
 (or  $(-\infty, -2) \cup (-2, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ )

b) (1 point) Find the  $x$ -intercept(s) and  $y$ -intercept of  $f$  (if any).

$$x+1=0 \quad f(0) = \frac{1}{-2}$$

$$x=-1$$

$x$ -intercept(s):  $\underline{-1}$

$y$ -intercept:  $\underline{-\frac{1}{2}}$

c) (3 points) Find all vertical asymptotes of  $f$ . Also, be sure to show your analysis on each side of each vertical asymptote (you can write these as in class—for example: “As  $x \rightarrow 1^-$ ,  $y \rightarrow \infty$ ”).

Vertical asymptotes:  $\underline{x = \frac{1}{2}, x = -2}$

As  $x \rightarrow \frac{1}{2}^-$ ,  $y \rightarrow \underline{-\infty}$

$\frac{x+1}{(2x-1)(x+2)}$   
 $\frac{\text{top} \rightarrow \frac{3}{2}}{\text{bot} \rightarrow 0, \text{neg}}$

As  $x \rightarrow \frac{1}{2}^+$ ,  $y \rightarrow \underline{\infty}$

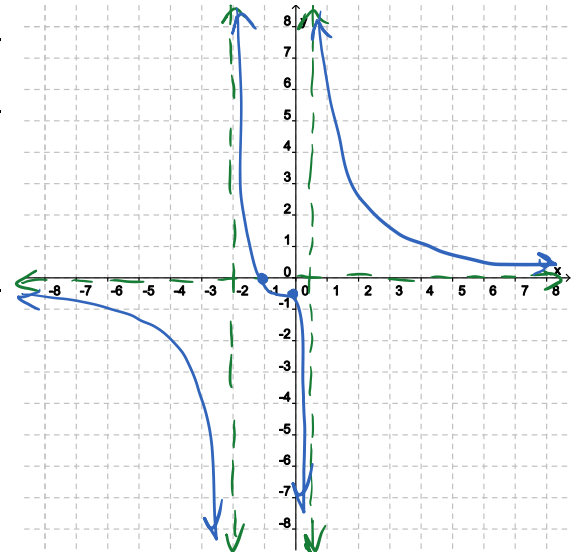
$\frac{\text{top} \rightarrow \frac{3}{2}}{\text{bot} \rightarrow 0, \text{pos}}$

As  $x \rightarrow -2^-$ ,  $y \rightarrow \underline{-\infty}$

$\frac{\text{top} \rightarrow -1}{\text{bot} \rightarrow 0, \text{pos}}$

As  $x \rightarrow -2^+$ ,  $y \rightarrow \underline{\infty}$

$\frac{\text{top} \rightarrow -1}{\text{bot} \rightarrow 0, \text{neg}}$



d) (1 point) Find the horizontal asymptote of  $f$ .

Horizontal asymptote:  $y = \underline{0}$

e) (2 points) Sketch the graph of  $f$ . Be sure to draw all asymptotes.

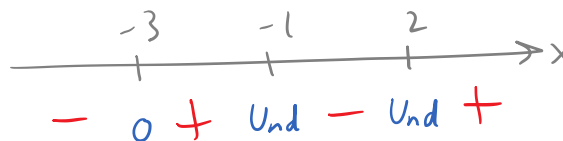
8. (2 points) Solve  $\frac{x+3}{x^2-x-2} < 0$ .

$$\frac{x+3}{(x+1)(x-2)} < 0$$

$\leftarrow -3$   
 $\uparrow -1 \quad \uparrow 2$

Answer:  $\underline{(-\infty, -3) \cup (-1, 2)}$

$$\frac{x+3}{(x+1)(x-2)}$$



9. (0 points) How many hours of sleep did you get last night?  $\underline{-\infty}$

Note: Be sure to double-check your work. And don't forget to turn in your homework! ☺