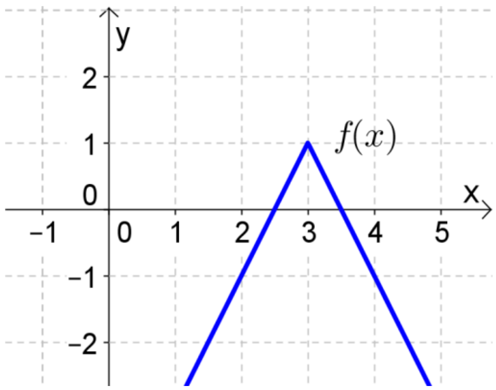
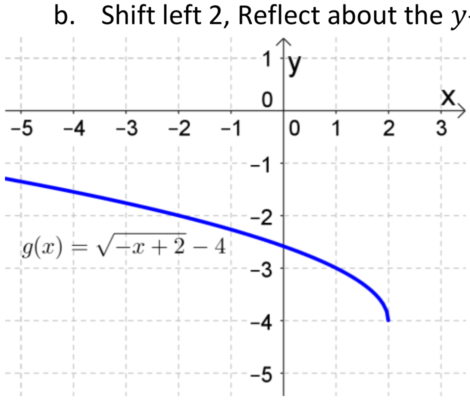
# Math 160 – Final Exam Review Exercises Answer Key

**Note:** Where possible, I wrote brief descriptions of reasoning—hopefully this provides a little more than just an answer. Please let me know if you find any mistakes or have any questions! - Prof. Beydler

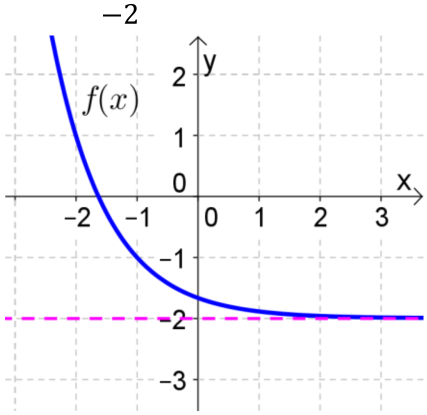
1. $-\frac{2}{(x-3)(x+h-3)}$
2. 
   a. Increasing: $[1,3]$, Decreasing: $(-\infty, 1]$ and $[3, \infty)$
   b. Local max $f(3) = 4$, Local min $f(1) = 1$
   c. $(-\infty, \infty)$
   d. $(-\infty, \infty)$
3. 
   a. Shift right 3, Stretch vertically by factor of 2, Reflect about $x$-axis, Shift up 1; Domain: $(-\infty, \infty)$, Range: $(-\infty, 1]$



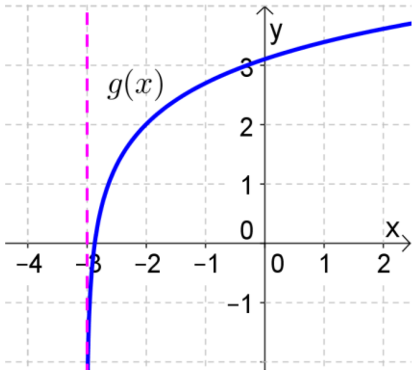

   b. Shift left 2, Reflect about the $y$-axis, Shift down 4; Domain: $(-\infty, 2]$, Range: $[-4, \infty)$



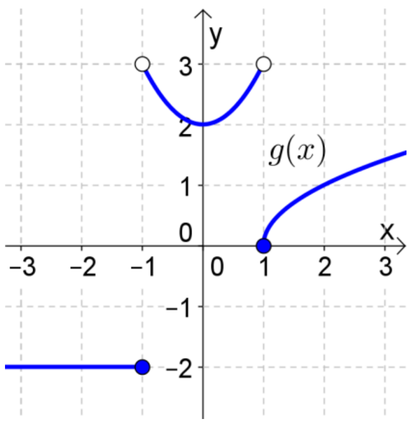

   c. Reflect about $y$-axis, Shift left 1, Shift down 2; Domain: $(-\infty, \infty)$, Range: $(-2, \infty)$, Asymptote: $y = -2$



d. Shift left 3, Shift up 2; Domain: $(-3, \infty)$, Range: $(-\infty, \infty)$, Asymptote: $x = -3$



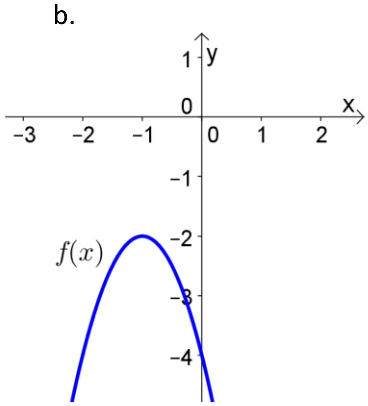

4.




5.

a. $(-1, -2)$

b.




c. Maximum value $-2$.

d. $(-\infty, -2]$

6. $C(x) = 4\left(2\left(\frac{1200}{x}\right) + x\right) + 6x = \mathbf{\frac{9600}{x} + 10x}$

7. 1000 ft$^2$ (Note: As a function of $x$, the area is $A(x) = 160x - \frac{32}{5}x^2$.)

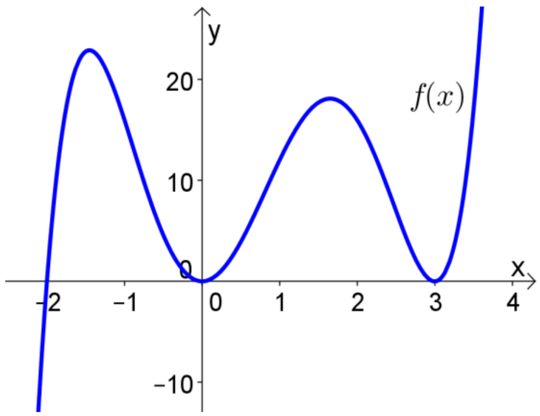
8.

a. $x$-intercepts: $0, -2, 3$; $y$-intercept: $0$

b. $(-3, -324), (-1, 16), (1,12), (4, 96)$ (Answers may vary.)

c. As $x \to -\infty$, $f(x) \to -\infty$. As $x \to \infty$, $f(x) \to \infty$.

d.




9. $x^2 - 2 + \frac{x-5}{2x^2+1}$

10.

a. Complete factorization: $(2x-1)^2(x+1)(x-3)$; Zeros: $\frac{1}{2}$ (multiplicity 2), $-1$, $3$

b. Complete factorization: $(x-2)(x-(2+i))(x-(2-i))$; Zeros: $2$, $2 \pm i$

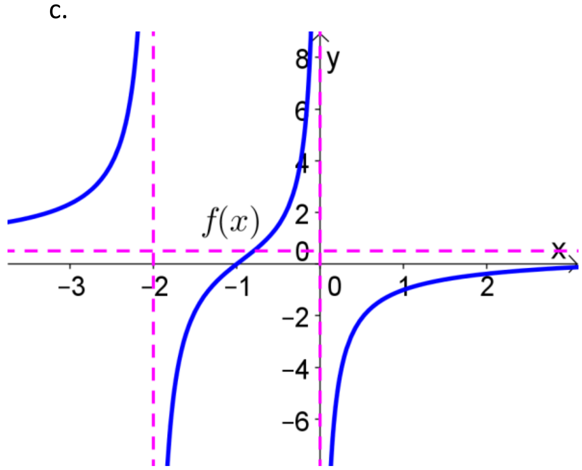
11. $2x^4 - 11x^3 + 13x^2 + 16x - 10$

12.

a. $x$-intercepts: $-1$, $4$; $y$-intercept: none

b. Vertical asymptotes: $x = -2$ and $x = 0$; Horizontal asymptote: $y = \frac{1}{2}$; Slant asymptote: none

c.

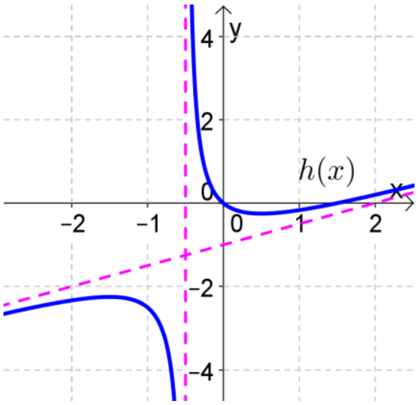


13.

a. $x$-intercepts: $0, \frac{3}{2}$; $y$-intercept: $0$

b. Vertical asymptote: $x = -\frac{1}{2}$; Horizontal asymptote: none; Slant asymptote: $y = \frac{1}{2}x - 1$

c.




14.

a. $(-\infty, -3] \cup [-2, 2]$

b. $(-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$

15. $f^{-1}(x) = \frac{x+2}{3-2x}$. Domain: $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

16.

a. $x = \frac{\ln(1/2)}{2}$

b. $x = \pm 5$

c. $x = \frac{2 \ln 5}{\ln 7 + 2 \ln 5}$

d. $x = \frac{-1+\sqrt{4e+9}}{2} \approx 1.729$ (Note: $x = \frac{-1-\sqrt{4e+9}}{2}$ does not solve the original equation)

e. $x = 4$

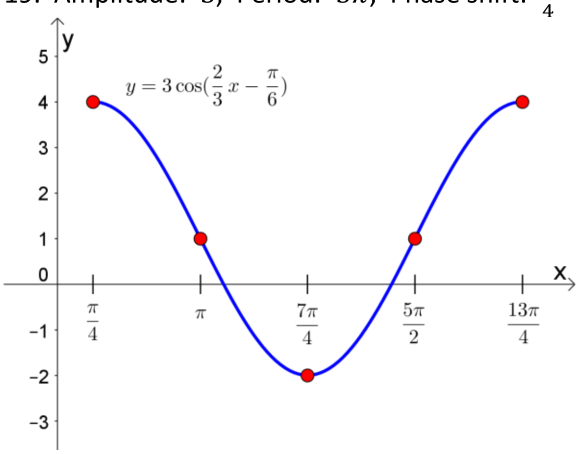
17.

a. 1,755,377 turkeys (Note: $k = \frac{\ln 2}{7} \approx 0.099021$, $N(t) = 45000e^{0.099021t}$)

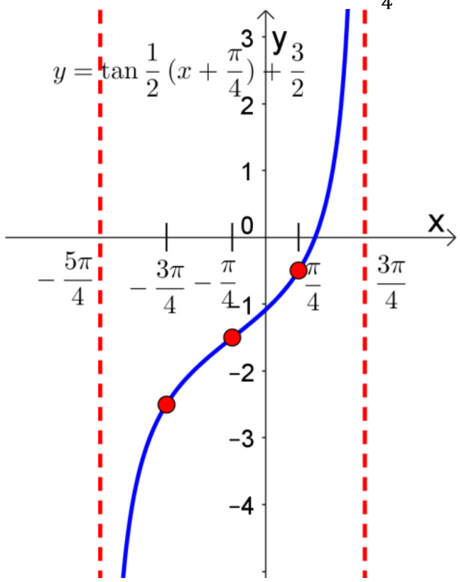
b. $t = \frac{\ln\left(\frac{200}{9}\right)}{0.099021} \approx 31.32$ years, so in 2044

18. 12.76 minutes after the initial temperature reading (Note: $k = \frac{\ln\left(\frac{115}{185}\right)}{-5} \approx 0.095085$, $T(t) = 65 + 185e^{-0.095085t}$, answer is $\frac{\ln\left(\frac{55}{185}\right)}{-0.095085}$)

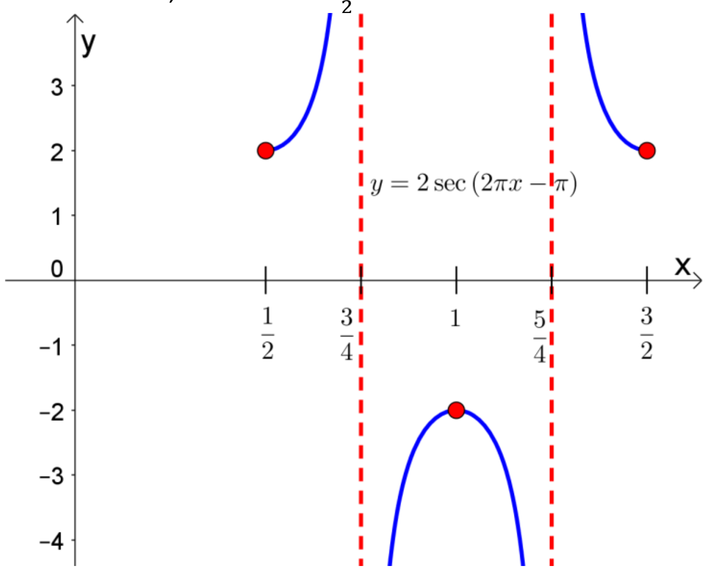
19. Amplitude: 3, Period: $3\pi$, Phase shift: $\frac{\pi}{4}$



20. Period: $2\pi$, Phase shift: $-\frac{\pi}{4}$




21. Period: 1, Phase shift: $\frac{1}{2}$




22.

a. $-\frac{\pi}{4}$

b. $\frac{25}{24}$

23.

a. $\frac{1}{\sqrt{x^2+1}}$

b. $\frac{xy+\sqrt{1-x^2}\sqrt{1-y^2}}{y\sqrt{1-x^2}-x\sqrt{1-y^2}}$

c. $2x\sqrt{1-x^2}$

24.

a. $LHS = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x(1+\sin x)}{\cos x(1+\sin x)} + \frac{\cos^2 x}{\cos x(1+\sin x)} = \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)} = \frac{(\sin x+1)}{\cos x(1+\sin x)} = \frac{1}{\cos x} = \sec x = RHS$

b. $RHS = \frac{\sin(x-y)}{\cos x\cos y} = \frac{\sin x\cos y-\cos x\sin y}{\cos x\cos y} = \frac{\sin x\cos y}{\cos x\cos y} - \frac{\cos x\sin y}{\cos x\cos y} = \frac{\sin x}{\cos x} - \frac{\sin y}{\cos y} = \tan x - \tan y = LHS$

25. $\sqrt{\frac{3-2\sqrt{2}}{6}}$

26. $\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

27.

a. All solutions: $\pi k, \frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k$; Solutions in $[0, 2\pi)$: $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

b. All solutions: $\frac{\pi}{2} + 2\pi k, \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k$; Solutions in $[0, 2\pi)$: $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

c. All solutions: $\pi k, \frac{5\pi}{18} + \frac{2\pi}{3}k, \frac{7\pi}{18} + \frac{2\pi}{3}k$; Solutions in $[0, 2\pi)$: $0, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \pi, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$

28.

a. $\sqrt{34}$

b. $-26.57°$

c. $\cos^{-1}\left(\frac{4}{\sqrt{65}}\right) \approx 60.26°$

d. $\frac{4}{\sqrt{13}}$

e. $\langle \frac{12}{13}, \frac{8}{13} \rangle$

f. $\overrightarrow{u_1} = \langle \frac{12}{13}, \frac{8}{13} \rangle$ and $\overrightarrow{u_2} = \langle \frac{14}{13}, -\frac{21}{13} \rangle$

29. No, since $\vec{u} \cdot \vec{v} = 1 \neq 0$.

30. Ground speed: 137.48 mi/h, Direction: $S\ 20.89°\ E$ (Note: plane's velocity relative to ground is approximately $\langle 49.03, -128.44 \rangle$)

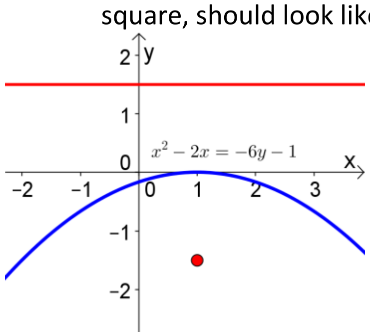
31. $S\ 16.26°\ W$

32. Magnitude of force required to keep it from sliding down ramp: 355.88 lbs. Magnitude of force experienced by ramp due to weight of book: 1328.15 lbs.

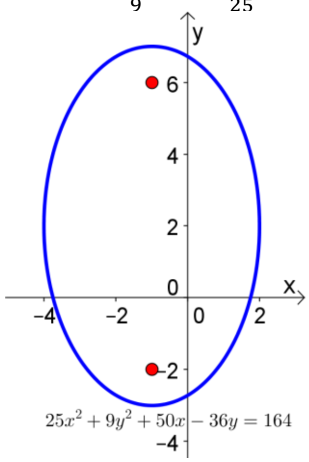
33. $7.66°$

34.

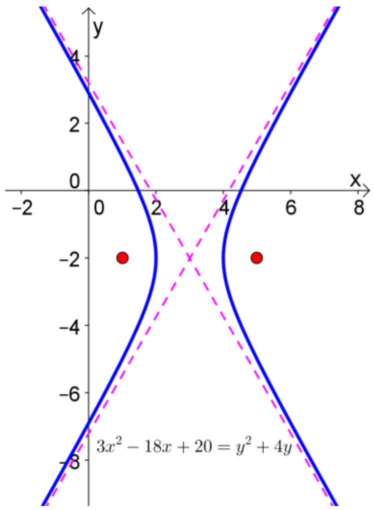
a. Parabola, Vertex: $(1,0)$, Focus: $\left(1, -\frac{3}{2}\right)$, Directrix: $y = \frac{3}{2}$, Focal diameter: 6 (Note: after completing square, should look like $(x-1)^2 = -6y$)




b. Ellipse, Center: $(-1,2)$, Foci: $(-1, 6)$ and $(-1, -2)$, Vertices: $(-1, 7)$ and $(-1, -3)$, Major axis length: 10, Minor axis length: 6, Eccentricity: $\frac{4}{5}$ (Note: after completing square, should look like $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$)



c. Hyperbola, Center: $(3,-2)$, Foci: $(5,-2)$ and $(1,-2)$, Vertices: $(4,-2)$ and $(2,-2)$, Asymptotes: $y+2=\pm\frac{\sqrt{3}}{1}(x-3)$ which become $y=\sqrt{3}x-3\sqrt{3}-2$ and $y=-\sqrt{3}x+3\sqrt{3}-2$ (Note: after competing square, should look like $\frac{(x-3)^2}{1}-\frac{(y+2)^2}{3}=1$)




35. $(x-3)^2=-12(y+1)$
36. $\frac{(x-1)^2}{5}+\frac{y^2}{9}=1$
37. $\frac{x^2}{9}-\frac{4y^2}{9}=1$
38. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}+\frac{D}{(2x+1)^2}+\frac{Ex+F}{2x^2+1}+\frac{Gx+H}{(2x^2+1)^2}$
39. $-\frac{1}{x}+\frac{2}{x-3}-\frac{3}{(x-3)^2}$
40. $\frac{2}{x-1}+\frac{3x}{x^2+4}$
41. $(-1,0)$ and $(2,\pm\sqrt{3})$
42. $(-\frac{1}{3},-\frac{11}{9})$ and $(1,-3)$
43. $a_1=0,\ a_2=1,\ a_3=3,\ a_4=64$
44. $-949$
45. $a_n=\frac{(-1)^n3^{n-1}}{2^n}$ (or $a_n=-\frac{1}{2}\left(-\frac{3}{2}\right)^{n-1}$)
46. $a_n=\frac{(-1)^{n-1}n^3\ln(3n-1)}{4n-1}$
47. $\sum_{k=1}^{9}\frac{(-1)^{k-1}\sqrt{2k-1}}{2k+1}$
48. Common difference: 3, $a_{100}=293$
49. Common ratio: $-\frac{2}{5}$, $a_n=-2\cdot\left(-\frac{2}{5}\right)^{n-1}$
50. $\frac{4039}{6561}$
51. 2415
52.
    a. Convergent (since $r=-\frac{1}{3}$, so $-1<r<1$). The sum is $-\frac{9}{4}$.
    b. Divergent (since $r=-\frac{5}{4}$, so $r\le -1$).
53.

Base case ($n=1$): $1=\frac{1\cdot(3\cdot1-1)}{2}\Rightarrow 1=1$ ✓

Induction step: Assume $n=k$ case is true: $1+4+7+\cdots+(3k-2)=\frac{k(3k-1)}{2}$.

(Show that $n=k+1$ case is true: $1+4+7+\cdots+(3k-2)+(3(k+1)-2)=\frac{(k+1)(3(k+1)-1)}{2}$)

$1 + 4 + 7 + \cdots + (3k - 2) + (3(k+1) - 2)$
$= \frac{k(3k-1)}{2} + (3(k+1) - 2)$
$= \frac{k(3k-1)}{2} + (3k + 3 - 2)$
$= \frac{k(3k-1)}{2} + (3k + 1)$
$= \frac{k(3k-1)}{2} + \frac{6k+2}{2}$
$= \frac{k(3k-1)+6k+2}{2}$
$= \frac{3k^2-k+6k+2}{2}$
$= \frac{3k^2+5k+2}{2}$
$= \frac{(k+1)(3k+2)}{2}$
$= \frac{(k+1)(3(k+1)-1)}{2}$ ✓
Thus, by induction, $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$ for all natural numbers $n$.
54.
Base case ($n = 1$): $\frac{1}{1\cdot 2} = \frac{1}{1+1} \Rightarrow \frac{1}{2} = \frac{1}{2}$ ✓
Induction step: Assume $n = k$ case is true: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k\cdot(k+1)} = \frac{k}{k+1}$.
(Show that $n = k + 1$ case is true: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k\cdot(k+1)} + \frac{1}{(k+1)\cdot((k+1)+1)} = \frac{k+1}{(k+1)+1}$)
$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k\cdot(k+1)} + \frac{1}{(k+1)\cdot((k+1)+1)}$
$= \frac{k}{k+1} + \frac{1}{(k+1)\cdot((k+1)+1)}$
$= \frac{k}{k+1} + \frac{1}{(k+1)\cdot(k+2)}$
$= \frac{k\cdot(k+2)}{(k+1)\cdot(k+2)} + \frac{1}{(k+1)\cdot(k+2)}$
$= \frac{k\cdot(k+2)+1}{(k+1)\cdot(k+2)}$
$= \frac{k^2+2k+1}{(k+1)\cdot(k+2)}$
$= \frac{(k+1)^2}{(k+1)\cdot(k+2)}$
$= \frac{k+1}{k+2}$
$= \frac{k+1}{(k+1)+1}$ ✓
Thus, by induction, $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n\cdot(n+1)} = \frac{n}{n+1}$ for all natural numbers $n$.
55. $32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$
56. $1 - \frac{15}{x} + \frac{105}{x^2} - \cdots$
57. $3360$