# Math 160 – Final Exam Info and Review Exercises

Fall 2018, Prof. Beydler

## Test Info

- Will cover almost all sections in this class.
- This will be a 2-part test. Part 1 will be **no calculator**. Part 2 will be **scientific calculator only**.
- No notes, no books, no phones during the final exam. Please don't fail the class because of a phone in your lap!
- As usual, there will be a seating chart for the final exam.
- Where to get help as you're studying:
    - Office hours
    - TMARC, LAC, or other tutoring centers
    - E-mail me at dbeydler@mtsac.edu
- If you go to a Mt. SAC tutoring center for 4 hours between Test #3 and the Final Exam, you'll get 1% extra credit towards the Final Exam.

### **Not on the test:**

- Determining if a function is even or odd (or neither). (Part of packet #2)
- Knowing how to compose two functions and determining the resulting domains. (Part of packet #12)
- Expanding and combining expressions using the Laws of Logarithms. (Part of packet #14)
- Using the Change of Base Theorem. (Part of packet #14)
- Finding exact values of trig functions at given angles. (Part of packet #17)

## Review Exercises

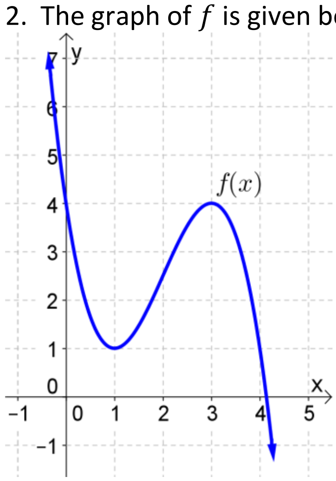
**Note:** If you write up solutions to all of the review exercises listed below, and hand them in at the test, you can earn up to 2% extra credit towards your test! It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the test. Please review the notes, previous quizzes, and homework problems to fully prepare for the test.

Types of problems that will appear on Part 1 are labeled **NC** (for **N**o **C**alculator).

### FUNCTIONS

1. Suppose $f(x) = \frac{2}{x-3}$. Find $\frac{f(x+h)-f(x)}{h}$ and simplify by canceling the factor of $h$. (**NC**)

2. The graph of $f$ is given below. (**NC**)




   a) Determine the intervals on which $f$ is increasing and decreasing.

   b) Find all local maxima and minima. Write your answers in the form $f(123) = 456$.

   c) Find the domain of $f$.

   d) Find the range of $f$.

3. Graph the following functions. Be sure to describe the transformations to the basic function. State the domain and range of each. (**NC**)

a) $f(x) = 1 - 2|x - 3|$

b) $g(x) = \sqrt{-x + 2} - 4$

c) $f(x) = 3^{-(x+1)} - 2$ (Also, find the asymptote.)

d) $g(x) = 2 + \ln(x + 3)$ (Also, find the asymptote.)

4. Graph $g(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ x^2 + 2 & \text{if } -1 < x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$ (**NC**)

5. Let $f(x) = -2x^2 - 4x - 4$.

a) Find the vertex of $f$.

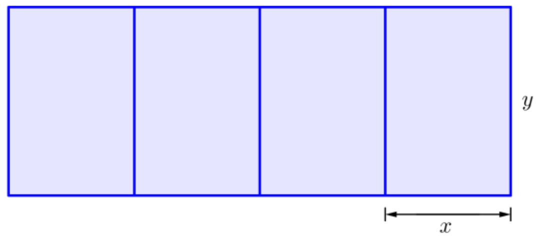
b) Sketch the graph of $f$. Be sure to plot the vertex and $y$-intercept.

c) Find the maximum or minimum value of $f$.

d) Find the range of $f$.

6. You want to fence a rectangular garden with one side against a road. Most of the fencing costs $4 per foot, but the fencing next to the road must be sturdier and costs $6 per foot. You want to have an area of 1200 ft$^2$. Find a function in one variable that models the cost of fencing the garden.

7. A rectangular plot of land is going to be divided into four adjacent, equal-sized playgrounds (see diagram). If the total number of feet of fencing is 200, find the maximum total area enclosed by all four playgrounds. (Note: there is only one fence between each pair of playgrounds, not two.)




8. Let $f(x) = x^2(x+2)(x-3)^2$.

a) Find the $x$-intercept(s) and the $y$-intercept of $f$.

b) Find a test point between the $x$-intercepts of $f$, as well as a test point before the first $x$-intercept, and a test point after the last $x$-intercept.

c) Determine the end behavior of $f$.

d) Sketch the graph of $f$.

9. Divide using long division: $\frac{2x^4-3x^2+x-7}{2x^2+1}$ (**NC**)

10. Find the complete factorization and all zeros of the following polynomials. (**NC**)
    a) $P(x) = 4x^4 - 12x^3 - 3x^2 + 10x - 3$

    b) $P(x) = x^3 - 6x^2 + 13x - 10$

11. Find a degree 4 polynomial with zeros $3+i$, $\frac{1}{2}$, and $-1$. (**NC**)

12. Let $f(x) = \frac{x^2-3x-4}{2x^2+4x}$
    a) Find the $x$-intercept(s) and $y$-intercept of $f$ (if any).

    b) Find all vertical, horizontal, and slant asymptotes of $f$. If none, write "none". Also, be sure to show your analysis on each side of each vertical asymptote (if any).

    c) Sketch the graph of $f$.

13. Let $h(x) = \frac{2x^2-3x}{4x+2}$

a) Find the $x$-intercept(s) and $y$-intercept of $h$ (if any).

b) Find all vertical, horizontal, and slant asymptotes of $h$. If none, write "none". Also, be sure to show your analysis on each side of each vertical asymptote (if any).

c) Sketch the graph of $h$.

14. Solve the following inequalities.

a) $x^5 + 3x^4 \leq 4x^3 + 12x^2$

b) $\frac{7x}{2x-1} > 3$

15. Find the inverse of $f(x) = \frac{3x-2}{2x+1}$. Be sure to state the domain of $f^{-1}(x)$. (**NC**)

## EXPONENTIAL AND LOG FUNCTIONS

16. Solve the following equations.

a) $2e^{4x} + 5e^{2x} - 3 = 0$

b) $x^2e^{4x} - 25e^{4x} = 0$

c) $7^{x/2} = 5^{1-x}$

d) $\ln(x-1)+\ln(x+2)=1$

e) $\log_5 x+\log_5(x+1)=\log_5 20$

17. The population of turkeys on an island was 45,000 in 2013, and the observed doubling time for the population of the turkeys is 7 years. (For this exercise, use the exponential growth model $N(t)=N_0e^{kt}$, and round the growth constant $k$ to 5 significant figures.)

a) What will the population of turkeys be in 2050?

b) In what year will the population of turkeys will reach 1,000,000?

18. You heat some water and steep some tea. The cup of tea is initially $250°F$. Five minutes later, the tea is $180°F$. Suppose the room temperature is $65°F$. First, use Newton's Law of Cooling to find a function that models the temperature of the cup of tea $t$ minutes after your initial temperature reading. Then, use the function to predict when the tea will be $120°F$. (For this exercise, use the model $T(t) = T_s + (T_0 - T_s)e^{-kt}$, and round $k$ to 5 significant figures.)

## TRIG FUNCTIONS

19. Find the amplitude, period, and phase shift of $y = 3\cos\left(\frac{2}{3}x - \frac{\pi}{6}\right) + 1$ and graph one complete period. Be sure to find the 5 key points.

20. Find the period and phase shift of $y = \tan\frac{1}{2}\left(x + \frac{\pi}{4}\right) - \frac{3}{2}$ and graph one complete period. Be sure to find the 5 key points/asymptotes.

21. Find the period and phase shift of $y = 2\sec(2\pi x - \pi)$ and graph one complete period. Be sure to find the 5 key points/asymptotes.

22. Find the exact value <u>without a calculator</u>. (**NC**)

a) $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$

b) $\csc\left(\cos^{-1}\frac{7}{25}\right)$

23. Write each of the following as an algebraic expression in $x$ (and possibly $y$). (**NC**)

a) $\cos(\tan^{-1} x)$ ($x$ is any real number)

b) $\tan(\sin^{-1} x + \cos^{-1} y)$ ($-1 \le x \le 1$ and $-1 \le y \le 1$)

c) $\sin(2\cos^{-1} x)$ $(-1 \leq x \leq 1)$

24. Prove the following identities. (**NC**)

a) $\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \sec x$

b) $\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$

25. If $\csc x = 3$ and $\frac{\pi}{2} < x < \pi$, find $\cos \frac{1}{2} x$. (**NC**)

26. Express $\sin^4 x$ in terms of the first power of cosine. (**NC**)

27. For each of the following trigonometric equations, first find all solutions (in radians). Then find all solutions (in radians) in the interval $[0,2\pi)$.

a) $\sin 2x = \sin x$

b) $2\cos^2 x \sin x - 2\cos^2 x - \sin x + 1 = 0$

c) $2\tan x \cos 3x = -\sqrt{3}\tan x$

## VECTORS

28. Suppose $\vec{u} = \langle 2, -1 \rangle$ and $\vec{v} = \langle 3, 2 \rangle$.
    a) Find $\|3\vec{u} - \vec{v}\|$.

    b) Find the direction (in degrees) of $\vec{u}$ from the positive $x$-axis.

    c) Find the angle (in degrees) between $\vec{u}$ and $\vec{v}$.

    d) Find the component of $\vec{u}$ along $\vec{v}$.

    e) Find the projection of $\vec{u}$ onto $\vec{v}$.

    f) Decompose $\vec{u}$ into $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$, where $\overrightarrow{u_1}$ is parallel to $\vec{v}$ and $\overrightarrow{u_2}$ is orthogonal to $\vec{v}$.

29. Are $\vec{u} = \langle 1,3 \rangle$ and $\vec{v} = \langle -2, 1 \rangle$ orthogonal? Show your reasoning. (**NC**)

30. A plane heads $S\ 10^\circ\ E$ at 150 mi/h. It experiences a 30 mi/h crosswind blowing $N\ 50^\circ\ E$. What is the ground speed of the plane? What is the plane's direction of movement relative to the ground? (Round all values to 2 decimal places.)

31. A swimmer is swimming 2.5 mi/h (relative to the water) across a straight river flowing east at 0.7 mi/h. In what direction should the swimmer head in order to arrive at a landing point that's due south of her?

32. A calculus book is on a ramp that is inclined $15^\circ$ to the horizontal. If the calculus book weighs 1375 lbs, find the magnitude of the force required to keep it from sliding down the ramp (ignoring friction). Also, find the magnitude of the force experienced by the ramp due to the weight of the calculus book.

33. A 15000-lb tractor is on a hill. If a force of 2000 lbs is just sufficient to keep the tractor from rolling down the hill, find the angle of inclination of the hill. (Assume no friction on the hill.)

## CONIC SECTIONS

34. Determine whether the equation represents a parabola, an ellipse, or a hyperbola. If the graph is a parabola, find the vertex, focus, directrix, and focal diameter. If it is an ellipse, find the center, foci, vertices, lengths of the major and minor axes, and eccentricity. If it is a hyperbola, find the center, foci, vertices, and asymptotes. Then sketch the graph of the equation. (**NC**)

a) $x^2 - 2x = -6y - 1$

b) $25x^2 + 9y^2 + 50x - 36y = 164$

c) $3x^2 - 18x + 20 = y^2 + 4y$

35. Find an equation for the parabola that satisfies the given conditions. (**NC**)
Focus: $(3, -4)$, directrix: $y = 2$

36. Find an equation for the ellipse that satisfies the given conditions. (**NC**)
Foci: $(1, \pm 2)$, vertices: $(1, \pm 3)$

37. Find an equation for the hyperbola that satisfies the given conditions. (**NC**)
Vertices: $(\pm 3, 0)$, asymptotes: $y = \pm \frac{1}{2}x$

## Partial Fraction Decomposition

38. Write only the form of the partial fraction decomposition of $\frac{3x^2+2}{x^2(2x+1)^2(2x^2+1)^2}$ (**NC**)

39. Find the partial fraction decomposition of $\frac{x^2-3x-9}{x^3-6x^2+9x}$.

40. Find the partial fraction decomposition of $\frac{5x^2-3x+8}{x^3-x^2+4x-4}$.

## SYSTEMS OF NONLINEAR EQUATIONS

41. Find all solutions of the system: $\begin{cases} x^2 - y^2 = 1 \\ 2x^2 - y^2 = x + 3 \end{cases}$ (**NC**)

42. Find all solutions of the system: $\begin{cases} x^2 + 2y + 2x + 3 = 0 \\ 2x^2 + y + 1 = 0 \end{cases}$ (**NC**)

## SEQUENCES

43. Find the first four terms of the sequence defined recursively by:
$a_1 = 0,\ a_n = n^{a_{n-1}}$

44. Find the following sum.

$$\sum_{i=-1}^{4} (-i)^{i+1}$$

45. Find a formula for the $n$th term of the sequence $-\frac{1}{2}, \frac{3}{4}, -\frac{9}{8}, \frac{27}{16}, \ldots$ (**NC**)

46. Find a formula for the $n$th term of the sequence $\frac{\ln 2}{3}, -\frac{8 \ln 5}{7}, \frac{27 \ln 8}{11}, -\frac{64 \ln 11}{15}, \ldots$ (**NC**)

47. Write the sum using sigma notation. (**NC**)
$\frac{1}{3} - \frac{\sqrt{2}}{5} + \frac{\sqrt{4}}{7} - \frac{\sqrt{8}}{9} + \cdots + \frac{\sqrt{256}}{19}$

48. Find the common difference and the 100th term of the following arithmetic sequence.
$-4, -1, 2, 5, 8, \ldots$

49. Find the common ratio and the nth term of the following geometric sequence.
$-2, \frac{4}{5}, -\frac{8}{25}, \frac{16}{125} \ldots$

50. Find the sum: $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots+\frac{256}{6561}$

51. Find the sum: $-4+(-1)+2+5+8+\cdots+119$

52. Determine whether the infinite geometric series converges or diverges. If it converges, find its sum.

a) $-3+1-\frac{1}{3}+\frac{1}{9}-\cdots$

b) $3-\frac{15}{4}+\frac{75}{16}-\frac{375}{64}+\cdots$

## MATHEMATICAL INDUCTION

53. Prove that for every natural number $n$, $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$ (**NC**)

54. Prove that for every natural number $n$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$ (**NC**)

## Binomial Theorem

55. Use the Binomial Theorem to expand $(2x+3)^5$.

56. Use the Binomial Theorem to find the first three terms in the expansion of $\left(1-\frac{1}{x}\right)^{15}$.

57. Find the coefficient of $x^6$ in the expansion of $(x+2)^{10}$.