Math 160 – Final Exam Formulas

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This is a collection of some of the formulas and concepts that were covered in the class.
Please note that it does not include every formula/concept.

The "**Know the following formulas/concepts**" section has all of the formulas/concepts that you'll need to commit to memory and absorb into every fiber of your being.

The "**I'll give you the following formulas during the final exam**" section has, well, all of the formulas that I'll give you during the final exam.

Know the following formulas/concepts

Chapter 2

Average Rate of Change

The average rate of change of $f(x)$ from $x = a$ to $x = b$ is $\frac{f(b)-f(a)}{b-a}$

Transformations

For a function $f(x)$, and $c > 0$,
$f(x) + c$ shifts up
$f(x) - c$ shifts down
$cf(x)$ stretches vertically (if $c > 1$), or shrinks vertically (if $0 < c < 1$)
$-f(x)$ reflects about x-axis

$f(x - c)$ shifts right
$f(x + c)$ shifts left
$f(cx)$ stretches horizontally (if $0 < c < 1$), or shrinks horizontally (if $c > 1$)
$f(-x)$ reflects about y-axis

Inverse Functions

$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

How to find the inverse of $f(x)$:

1. Replace $f(x)$ with y.
2. Switch x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Chapter 3

Quadratic Functions ($f(x) = ax^2 + bx + c$)
Standard form: $f(x) = a(x-h)^2 + k$ (here, vertex at (h, k))
x-value of vertex: $x = -\frac{b}{2a}$

Polynomial Functions
If a factor $x - c$ has an **odd** multiplicity, then the curve will **pass through** the x-axis at $x = c$.
If a factor $x - c$ has an **even** multiplicity, then the curve will **"bounce" off** the x-axis at $x = c$:

Remainder Theorem
If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Factor Theorem
c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Rational Zeros Theorem
If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every rational zero of P is of the form $\frac{p}{q}$ where p is a factor of a_0 and q is a factor of a_n.

Fundamental Theorem of Algebra
Every polynomial with complex coefficients has at least one complex zero.

Complete Factorization Theorem
Every polynomial can be broken down into linear factors like this:
$P(x) = a(x - c_1)(x - c_2) \ldots (x - c_n)$ (here, $a, c_1, c_2, \ldots, c_n$ are complex numbers)

Zeros Theorem
Every degree n polynomial has exactly n zeros (here, a zero of multiplicity k is counted k times).

Conjugate Zeros Theorem
Complex zeros come in conjugate pairs. That is, if $a + bi$ is a zero, then $a - bi$ is also a zero.
(Note: This is only true for polynomials with real coefficients, which is mostly what you see anyway.)

Horizontal Asymptote of $\frac{P(x)}{Q(x)}$:
If deg(P) < deg (Q), then horizontal asymptote $y = 0$.
If deg(P) = deg(Q), then horizontal asymptote $y = \frac{\text{leading coefficient of } P}{\text{leading coefficient of } Q}$.
If deg(P) > deg(Q), then no horizontal asymptotes.

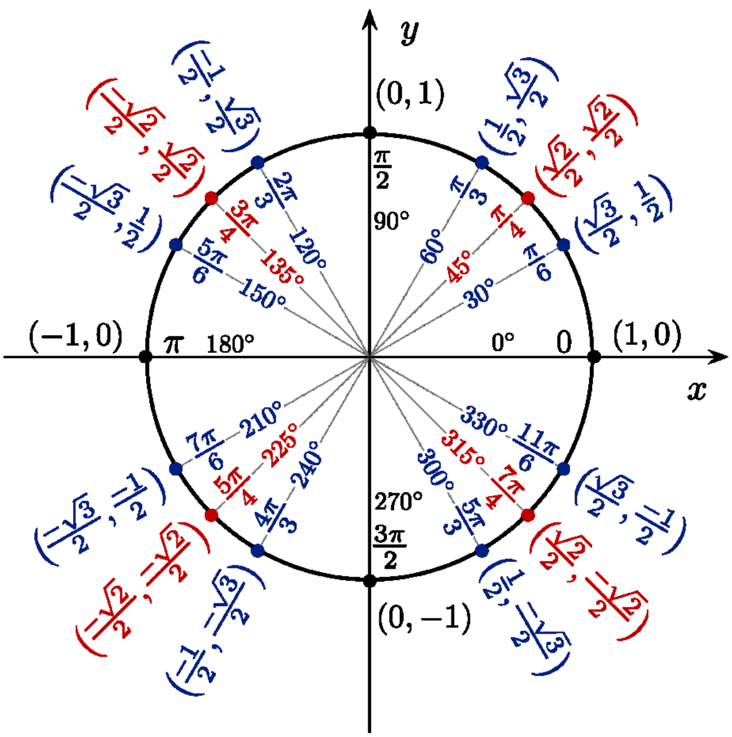
Chapter 4

$\log x = \log_{10} x$
$\ln x = \log_e x$

$a^{\log_a x} = x$
$\log_a a^x = x$

Laws of Logarithms
$\log_a(AB) = \log_a A + \log_a B$
$\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$
$\log_a A^C = C \log_a A$

Chapter 5



$\boldsymbol{y = a\sin k(x-b)}$ and $\boldsymbol{y = a\cos k(x-b)}$
Amplitude: $|a|$
Period: $\frac{2\pi}{k}$
Phase shift: b

$\boldsymbol{y = a\csc k(x-b)}$ and $\boldsymbol{y = a\sec k(x-b)}$
Period: $\frac{2\pi}{k}$

$\boldsymbol{y = a\tan k(x-b)}$ and $\boldsymbol{y = a\cot k(x-b)}$
Period: $\frac{\pi}{k}$

$\sin(\sin^{-1} x) = x$ for $-1 \le x \le 1$
$\sin^{-1}(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

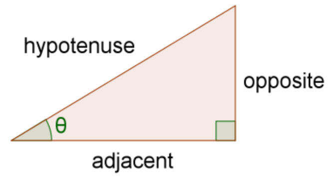
$\cos(\cos^{-1} x) = x$ for $-1 \le x \le 1$
$\cos^{-1}(\cos x) = x$ for $0 \le x \le \pi$

$\tan(\tan^{-1} x) = x$ for $-\infty < x < \infty$
$\tan^{-1}(\tan x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Chapters 6, 7

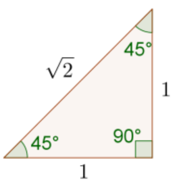
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$

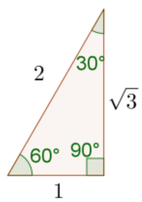
$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$



(opp = opposite, adj = adjacent, hyp = hypotenuse)

Special Triangles





Reciprocal Identities

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Even-Odd Identities

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$
$\cos(x - y) = \cos x \cos y + \sin x \sin y$

Double-Angle Formulas

$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

Power-Lowering Formulas

$\sin^2 x = \frac{1-\cos 2x}{2}$
$\cos^2 x = \frac{1+\cos 2x}{2}$

Half-Angle Formulas

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$
$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

Chapter 9

The magnitude (or length) of $\vec{v} = \langle a, b \rangle$ is $|\vec{v}| = \sqrt{a^2 + b^2}$
$\vec{\imath} = \langle 1,0 \rangle$ and $\vec{\jmath} = \langle 0,1 \rangle$
$\langle a, b \rangle = a\vec{\imath} + b\vec{\jmath}$

Dot Product
The dot product of $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$ is:
$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$

The Dot Product Theorem
$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$ (θ is the angle between $\vec{u}$ and $\vec{v}$)

Two nonzero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u} \cdot \vec{v} = 0$.

Chapter 10

Partial Fraction Decomposition
1. If we get a ***repeated*** **linear factor** $(\boldsymbol{ax + b})^{\boldsymbol{k}}$, then we'll have corresponding partial fractions:
   $\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$
2. If we get an **irreducible quadratic factor** $\boldsymbol{ax^2 + bx + c}$, then we'll have a corresponding partial fraction:
   $\frac{Ax+B}{ax^2+bx+c}$
3. If we get a ***repeated*** **irreducible quadratic factor** $(\boldsymbol{ax^2 + bx + c})^{\boldsymbol{k}}$, then we'll have corresponding partial fractions: $\frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$

If degree of top polynomial is ≥ degree of bottom polynomial, then divide first.

Chapter 11

Ellipses
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$c = \sqrt{a^2 - b^2}$ or (or if $b > a$, then $c = \sqrt{b^2 - a^2}$)

Horizontal Hyperbolas
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$c = \sqrt{a^2 + b^2}$
Asymptotes: $y = \pm\frac{b}{a}x$
Vertices: $(\pm a, 0)$

Vertical Hyperbolas
$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
Vertices: $(0, \pm b)$

Chapter 12

Arithmetic Sequences
$a, a+d, a+2d, a+3d, a+4d, \ldots$ (ex: $4, 7, 10, 13, 16, \ldots$)
$a_n = a + (n-1)d$

Geometric Sequences
$a, ar, ar^2, ar^3, ar^4, \ldots$ (ex: $2, 6, 18, 54, \ldots$)
$a_n = ar^{n-1}$

Infinite Geometric Series $(a + ar + ar^2 + ar^3 + \cdots)$
If $-1 < r < 1$, then the series converges to $\frac{a}{1-r}$
If $r \geq 1$ or $r \leq -1$, then the series diverges.

Mathematical Induction
1. **Base case**: Show that $P(1)$ is true.
2. **Induction step**: Assume $P(k)$ is true, and show that $P(k+1)$ is true (for any natural number k).

Binomial Theorem
$n! = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ ($n!$ is read "$\boldsymbol{n}$ **factorial**")
$0! = 1$

I'll give you the following formulas during the final exam

Chapter 4

Change of Base Theorem
$\log_b x = \frac{\log_a x}{\log_a b}$

Exponential Growth Model
$N(t) = N_0 e^{kt}$

Newton's Law of Cooling
$T(t) = T_s + (T_0 - T_s)e^{-kt}$

Chapters 6, 7

Trig Identities

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\tan\left(\frac{\pi}{2} - x\right) = \cot x$
$\csc\left(\frac{\pi}{2} - x\right) = \sec x$ $\sec\left(\frac{\pi}{2} - x\right) = \csc x$ $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ $\tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$ $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

Chapters 9

Components of Vectors

Component of $\vec{u}$ along $\vec{v}$ is $|\vec{u}|\cos\theta$ (θ is the angle between $\vec{u}$ and $\vec{v}$)

Component of $\vec{u}$ along $\vec{v}$ can also be calculated using: $\frac{\vec{\boldsymbol{u}}\cdot\vec{\boldsymbol{v}}}{|\vec{\boldsymbol{v}}|}$

Projections of Vectors

$\text{proj}_{\vec{v}}\vec{u} = (\text{component of } \vec{u} \text{ along } \vec{v})(\text{unit vector in direction of } \vec{v}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\right)\frac{\vec{v}}{|\vec{v}|} = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$

Chapter 11

Parabolas

$x^2 = 4py$ has focus $(0, p)$ and directrix $y = -p$.

$y^2 = 4px$ has focus $(p, 0)$ and directrix $x = -p$.

Focal diameter: $|4p|$

Chapter 12

Arithmetic Sequences

$S_n = \frac{(a_1 + a_n)\cdot n}{2}$

Geometric Sequences

$S_n = \frac{a(1-r^n)}{1-r}$

Binomial Coefficient: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Binomial Theorem: $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$

The term that contains a^{n-k} in the expansion of $(a+b)^n$ is: $\boldsymbol{\binom{n}{k}a^{n-k}b^k}$