

Dividing Polynomials

(covers parts of Sullivan 4.5)

Long Division

Ex 1.

Divide using long division: $\frac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$

Synthetic Division

Synthetic division is a quick way to divide by something of the form $x - c$.

Ex 2.

Divide using synthetic division: $\frac{x^4 - x^3 + x^2 - x + 2}{x - 2}$

Remainder Theorem:

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Why does this work?

So, instead of finding $P(c)$ by directly plugging in $x = c$, we can instead use synthetic division since the remainder will be $P(c)$ (by the Remainder Theorem).

Ex 3.

Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$. Use synthetic division and the Remainder Theorem to find $P(-2)$.

Factor Theorem:

c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

In other words, each zero corresponds with a factor, and each factor corresponds with a zero.

Ex 4.

Use the Factor Theorem to show that $x - 2$ is a factor of $P(x) = x^3 + 2x^2 - 3x - 10$.