1. Divide using long division: $\frac{18x^4+9x^2+3x+2}{3x^2+1}$

$$\begin{array}{r} 6x^2 \qquad\qquad +1 \\ 3x^2+1 \,\overline{)\, 18x^4+0x^3+9x^2+3x+2} \\ -(18x^4 \qquad\quad +6x^2) \\ \hline 3x^2+3x+2 \\ -(3x^2 \qquad +1) \\ \hline 3x+1 \end{array}$$

$$\frac{18x^4+9x^2+3x+2}{3x^2+1} = \boxed{6x^2+1+\frac{3x+1}{3x^2+1}}$$

2. Divide using synthetic division: $\frac{3x^3-2x^2+35}{x+2}$

$$\begin{array}{r|rrrr} -2 & 3 & -2 & 0 & 35 \\ & & -6 & 16 & -32 \\ \hline & 3 & -8 & 16 & 3 \end{array}$$

$$\frac{3x^3-2x^2+35}{x+2} = \boxed{3x^2-8x+16+\frac{3}{x+2}}$$

3. Is $x+1$ a factor of $x^{345}+3x^{172}-2x^{11}+7x^2-11$?

Let $f(x) = x^{345}+3x^{172}-2x^{11}+7x^2-11$.

$$f(-1) = (-1)^{345}+3(-1)^{172}-2(-1)^{11}+7(-1)^2-11$$

$$= -1+3+2+7-11$$

$$= 0$$

By the Factor Theorem, since $f(-1)=0$, $x+1$ is a factor of $f(x)$.

$\boxed{\text{Yes!}}$

Q: What goes around the world but stays in a corner?

4. Divide using long division: $\frac{3x^5-7x^3+5x^2+6x-6}{x^3-x+2}$

$\frac{3x^5}{x^3}$ $\qquad$ $\frac{-4x^3}{x^3}$

$$
\begin{array}{r}
3x^2 \qquad -4 \\
x^3+0x^2-x+2 \,\overline{)\, 3x^5+0x^4-7x^3+5x^2+6x-6} \\
-(3x^5+0x^4-3x^3+6x^2) \qquad\quad \\
\hline
-4x^3-x^2+6x-6 \\
-(-4x^3+0x^2+4x-8) \\
\hline
-x^2+2x+2
\end{array}
$$

So, $$\frac{3x^5-7x^3+5x^2+6x-6}{x^3-x+2} = \boxed{3x^2-4+\frac{-x^2+2x+2}{x^3-x+2}}$$

6. Divide using synthetic division: $\frac{2x^3-3x-5}{x+2}$

$$
\begin{array}{r|rrrr}
-2 & 2 & 0 & -3 & -5 \\
 & & -4 & 8 & -10 \\
\hline
 & 2 & -4 & 5 & -15
\end{array}
$$

$$\frac{2x^3-3x-5}{x+2} = \boxed{2x^2-4x+5-\frac{15}{x+2}}$$

8. Divide using synthetic division: $\frac{2x^3+3x^2-13x+5}{2x-3}$ (Hint: Factor out a $\frac{1}{2}$ first.)

$$\frac{2x^3+3x^2-13x+5}{2x-3} = \frac{1}{2}\left(\frac{2x^3+3x^2-13x+5}{x-3/2}\right)$$

$$
\begin{array}{r|rrrr}
3/2 & 2 & 3 & -13 & 5 \\
 & & 3 & 9 & -6 \\
\hline
 & 2 & 6 & -4 & -1
\end{array}
$$

$$= \frac{1}{2}\left(2x^2+6x-4-\frac{1}{x-3/2}\right)$$

$$= \boxed{x^2+3x-2-\frac{1}{2x-3}}$$

10. Let $P(x) = 2x^2 + 9x + 1$. Use synthetic division and the Remainder Theorem to find $P\left(\frac{1}{2}\right)$.

$$\begin{array}{r|rrr} \frac{1}{2} & 2 & 9 & 1 \\ & & 1 & 5 \\ \hline & 2 & 10 & 6 \end{array}$$

By the Remainder Theorem,

$$\boxed{P\left(\tfrac{1}{2}\right) = 6}$$

12. Use the Factor Theorem to show that $x + 3$ is a factor of $P(x) = x^4 + 3x^3 - 16x^2 - 27x + 63$.

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & -16 & -27 & 63 \\ & & -3 & 0 & 48 & -63 \\ \hline & 1 & 0 & -16 & 21 & 0 \end{array}$$

By the Remainder Theorem,

$P(-3) = 0$

Since $x = -3$ is a zero of $P(x)$, $x + 3$ is a factor of $P(x)$ (by the Factor Theorem). ∎