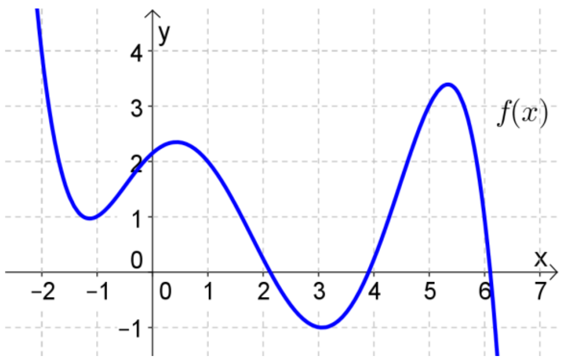
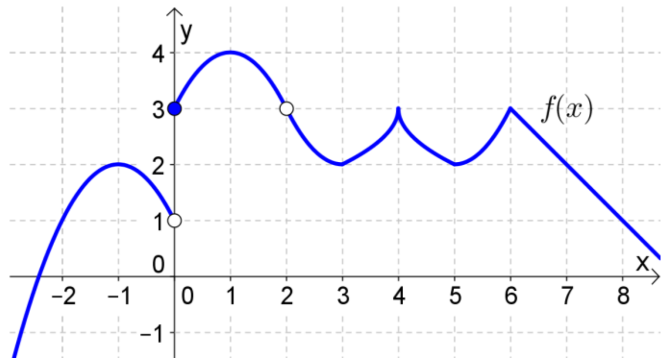
Polynomial Functions

(covers Sullivan 4.1)

A **polynomial function of degree n** is a function that can be written in the form:
$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

Polynomial functions are continuous and smooth. That means no gaps, holes, cusps, or corners.





The **end behavior** of a function means how the function behaves when $x \to \infty$ or $x \to -\infty$.
For non-constant polynomial functions, the end behavior is either $y \to \infty$ or $y \to -\infty$.

The highest degree term of a polynomial, called the _______________, determines its end behavior.

Ex 1.
Determine the end behavior of the polynomial $P(x) = -2x^4 + 5x^3 + 4x - 7$.

Ex 2.
Determine the end behavior of the polynomial $P(x) = 3x^5 - 2x^2 + x - 12212012$.

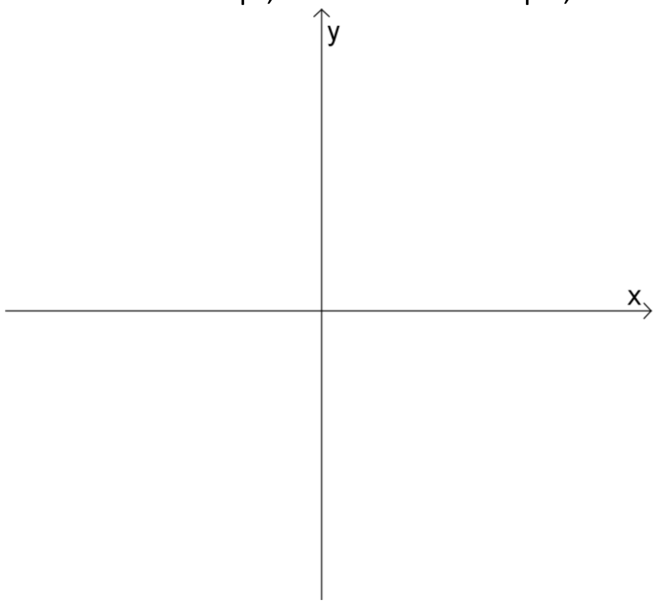
Note: Zeros of a polynomial correspond with factors, and visually mean x-intercepts.
ex: If $f(x) = x^2 - 5x + 6$, then since $f(2) = 0$, we must have a factor of $x - 2$. Also, there will be an x-intercept at $x = 2$.

Graphing Polynomial Functions

1. Factor to find zeros and plot **x-intercepts**.
2. Plot **test points** (before smallest x-intercept, between x-intercepts, and after largest x-intercept).
3. Determine **end behavior**.
4. **Graph**.

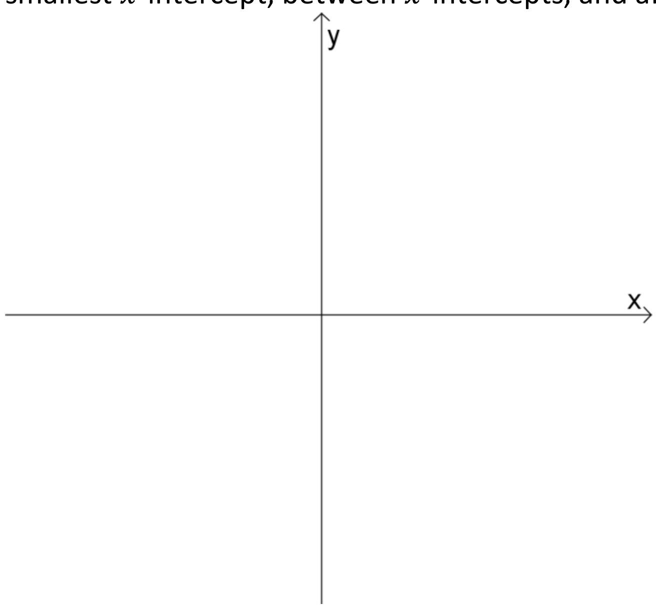
Ex 3.

Sketch the graph of $f(x) = -x^3 + 2x^2 + 3x$. Be sure to show intercepts, test points (before smallest x-intercept, between x-intercepts, and after largest x-intercept), and end behavior.



Ex 4.

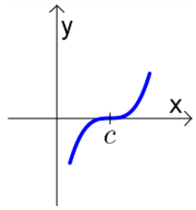
Sketch the graph of $f(x) = 2(x-3)^2(x+1)^2$. Be sure to show intercepts, test points (before smallest x-intercept, between x-intercepts, and after largest x-intercept), and end behavior.

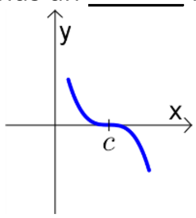


Multiplicity

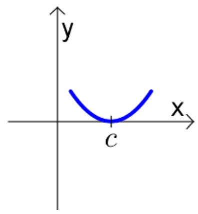
For the polynomial $P(x) = 2(x-5)^3(x-1)^4$, the factor $x-5$ has multiplicity ___, and the factor $x-1$ has multiplicity ___.

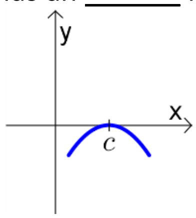
If a factor $x-c$ has an ______ multiplicity, then the curve will ______________ the x-axis at $x = c$:





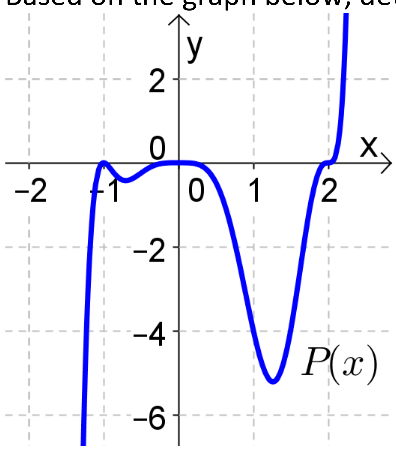
If a factor $x-c$ has an ______ multiplicity, then the curve will ______________ the x-axis at $x = c$:





Ex 5.

Based on the graph below, determine if the multiplicities of each zero of $P(x)$ are even or odd.



Note: Every polynomial of degree n has at most $n-1$ **turning points**.
ex: $P(x) = 7x^4 + 3x^3 - 5x^2 + 4x - 1$ has at most 3 turning points.

Also, if a polynomial's graph has n turning points, then the polynomials' degree is at least $n+1$.
ex: A polynomial with 5 turning points is at least degree 6.

Ex 6.

What is the least degree that the polynomial $P(x)$ shown below could have?