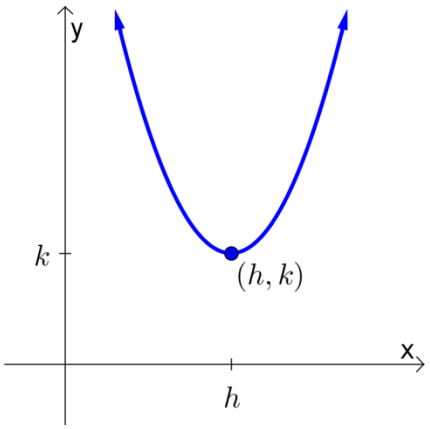
Quadratic Functions

(covers parts of Sullivan 3.3 and 3.4)

A _____ is a function that can be written in the form:
$f(x) = ax^2 + bx + c$ (where $a \neq 0$)

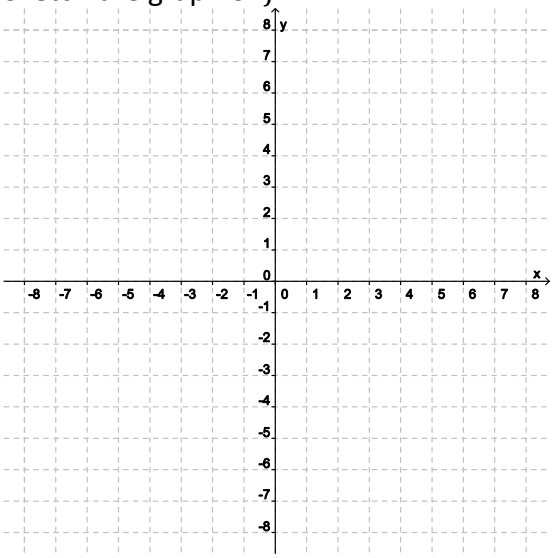


The graph of any quadratic function is a **parabola**.

Often, it is useful to write quadratic functions in **standard form**:
$f(x) = a(x - h)^2 + k$
It's useful because we can easily identify the parabola's vertex (h, k).
For both forms above, if $a > 0$ then the parabola opens ____, and if $a < 0$ then the parabola opens _____.

Ex 1.
Let $f(x) = 3x^2 + 6x + 5$. Express f in standard form (by completing the square).

What is the vertex of f? _____
Sketch the graph of f.



What is the minimum value of f? _____
What is the range of f? _____

Ex 2.

Express $f(x) = -2x^2 + 12x - 7$ in standard form (by completing the square).

Let's complete the square to get the vertex of a general quadratic function $f(x) = ax^2 + bx + c$.

$$\begin{aligned} f(x) &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \end{aligned}$$

So, the vertex is at $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$.

Now we have a shortcut formula for finding the x-value of the vertex: $\boldsymbol{x = -\frac{b}{2a}}$

Ex 3.

Let $f(x) = -2x^2 + 4x - 5$. Find the vertex of f.

Find the maximum or minimum value of f. State whether it is a maximum or minimum value.

What is the range of f? _____

Ex 4.

Bob has 1200 ft of fencing to fence in a rectangular garden. Find a function that models the area of the garden in terms of its width x. Then find the dimensions that maximize the area of the garden. Then find the maximum area of the garden.