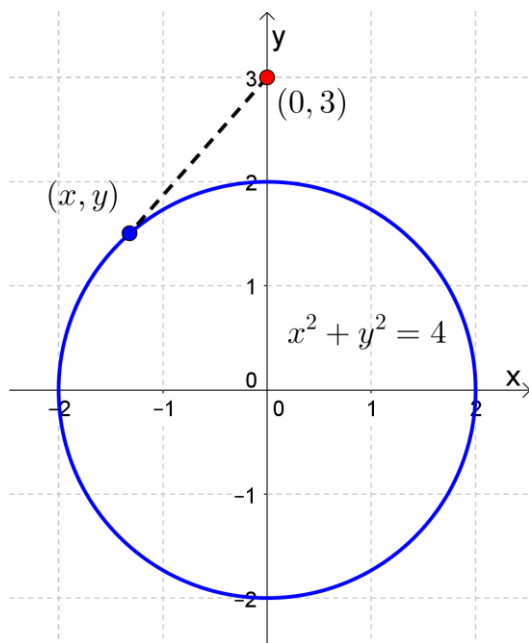


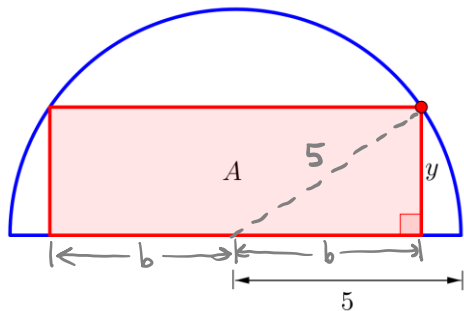
1. Find the distance from the point  $(0, 3)$  to the circle  $x^2 + y^2 = 4$  as a function of  $y$  only. Be sure to simplify your function.



$$\begin{aligned}
 d(x, y) &= \sqrt{(x-0)^2 + (y-3)^2} \\
 &= \sqrt{x^2 + (y-3)^2} \\
 d(y) &= \sqrt{4 - \cancel{y^2} + \cancel{y^2} - 6y + 9} \\
 &= \boxed{\sqrt{13 - 6y}}
 \end{aligned}$$

$x^2 + y^2 = 4$   
 $x^2 = 4 - y^2$

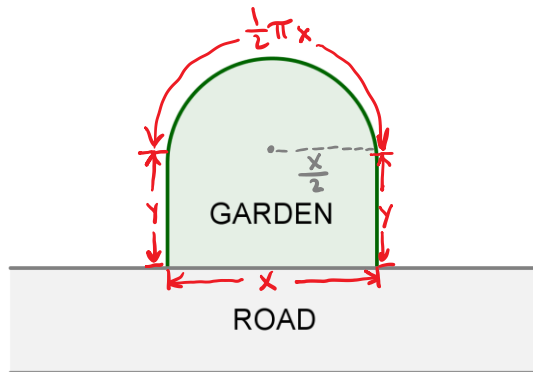
2. A rectangle is inscribed in a semicircle of radius 5. Find a function that models the area  $A$  of the rectangle in terms of its height  $y$ .



$$\begin{aligned}
 A(b, y) &= 2by \\
 A(y) &= \boxed{2y\sqrt{25 - y^2}}
 \end{aligned}$$

$b^2 + y^2 = 25$   
 $b = \sqrt{25 - y^2}$

4. You want to fence a garden with one side against a straight road. The garden consists of a rectangular region and a semicircular region as shown below. Most of the fencing costs \$3 per foot, but the fencing next to the road must be sturdier and costs \$4 per foot. You want to have an area of 400 ft<sup>2</sup>. Find a function in one variable that models the cost of fencing the garden.



The length of the semicircle is:

$$\frac{1}{2} (2\pi r) = \pi r = \pi \left(\frac{x}{2}\right) = \frac{1}{2} \pi x$$

Area of semicircle is  $\frac{1}{2} \pi r^2 = \frac{1}{2} \pi \left(\frac{x}{2}\right)^2$

$$C(x, y) = 3 \cdot \left(\frac{1}{2} \pi x + y + y\right) + 4 \cdot x$$

$$= \frac{3}{2} \pi x + 6y + 4x$$

$$C(x) = \frac{3}{2} \pi x + 6 \left(\frac{400}{x} - \frac{1}{8} \pi x\right) + 4x$$

$$= \frac{3}{2} \pi x + \frac{2400}{x} - \frac{3}{4} \pi x + 4x$$

$$= \frac{3}{4} \pi x + \frac{2400}{x} + 4x$$

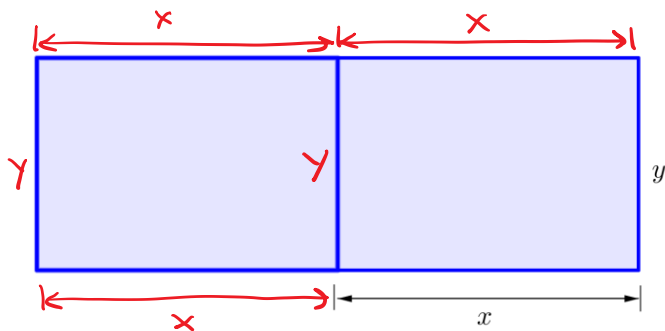
$A = 400$

$$xy + \frac{1}{2} \pi \left(\frac{x}{2}\right)^2 = 400$$

$$xy = 400 - \frac{1}{8} \pi x^2$$

$$y = \frac{400}{x} - \frac{1}{8} \pi x$$

6. A 10000-square-foot rectangular plot of land is going to be divided into two equal-sized, adjacent playgrounds (see diagram). Find the number of feet of fencing required as a function of  $x$ . (Note: here is only one fence between the playgrounds, not two.)



$$F(x, y) = 4x + 3y$$

$$F(x) = 4x + 3 \left(\frac{5000}{x}\right)$$

$$= 4x + \frac{15000}{x}$$

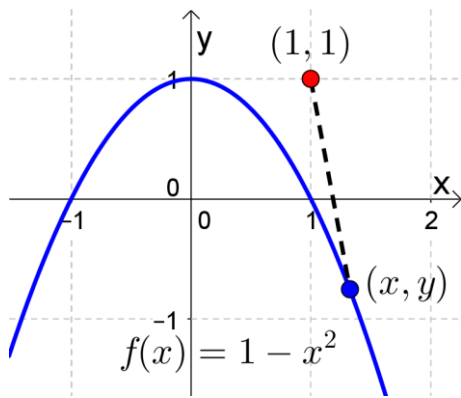
$A = 2x \cdot y$

$$10000 = 2xy$$

$$5000 = xy$$

$$y = \frac{5000}{x}$$

8. Find the distance from the point  $(1, 1)$  to the parabola  $f(x) = 1 - x^2$  as a function of  $x$  only. Be sure to simplify your function.



$$d(x, y) = \sqrt{(x-1)^2 + (y-1)^2} \quad \rightarrow y = 1 - x^2$$

$$d(x) = \sqrt{x^2 - 2x + 1 + (\cancel{1} - x^2 - \cancel{1})^2}$$

$$= \sqrt{x^2 - 2x + 1 + (-x^2)^2}$$

$$= \boxed{\sqrt{x^4 + x^2 - 2x + 1}}$$