Limits and More: A Calculus Preview!

(covers parts of Sullivan 14.1, 14.3, 14.4, and 14.5)

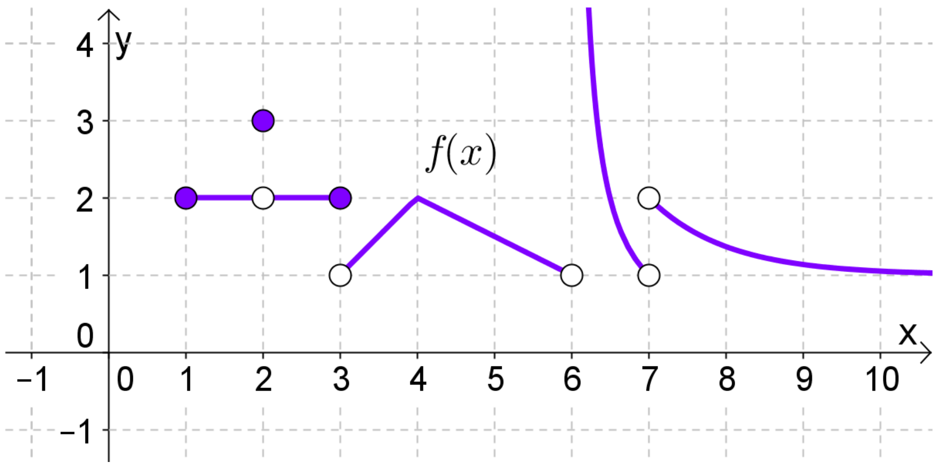
Graphically

Recall:

$x \to 2^-$ means ____________________________________

$x \to 2^+$ means ____________________________________

Ex 1.



Find the following. (Note: $\lim_{x\to 2} f(x)$ is read: "the limit of $f(x)$ as x approaches 2")

$\lim_{x\to 2^-} f(x)$ $\qquad$ $\lim_{x\to 2^+} f(x)$ $\qquad$ $\lim_{x\to 2} f(x)$ $\qquad$ $f(2)$

$\lim_{x\to 3^-} f(x)$ $\qquad$ $\lim_{x\to 3^+} f(x)$ $\qquad$ $\lim_{x\to 3} f(x)$ $\qquad$ $f(3)$

$\lim_{x\to 4^-} f(x)$ $\qquad$ $\lim_{x\to 4^+} f(x)$ $\qquad$ $\lim_{x\to 4} f(x)$ $\qquad$ $f(4)$

$\lim_{x\to 6^-} f(x)$ $\qquad$ $\lim_{x\to 6^+} f(x)$ $\qquad$ $\lim_{x\to 6} f(x)$ $\qquad$ $f(6)$

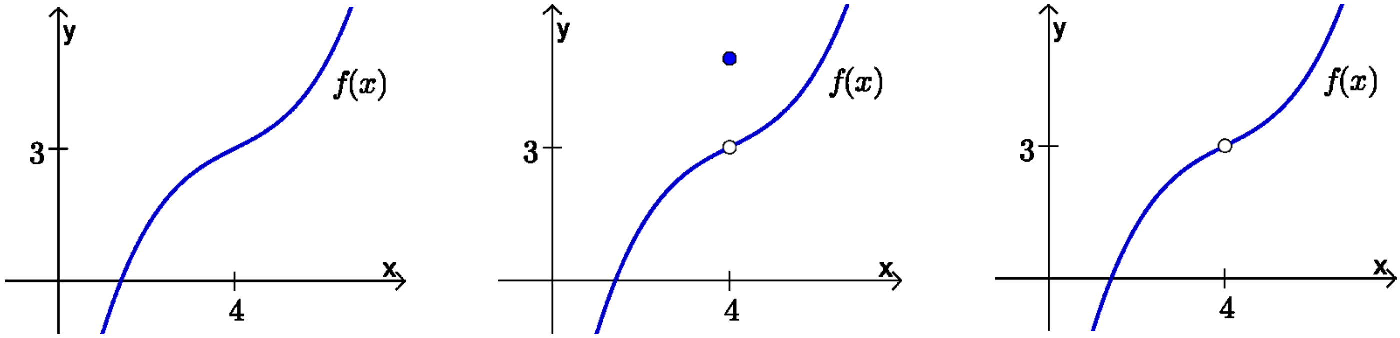
$\lim_{x\to 7^-} f(x)$ $\qquad$ $\lim_{x\to 7^+} f(x)$ $\qquad$ $\lim_{x\to 7} f(x)$ $\qquad$ $f(7)$

$\lim_{x\to \infty} f(x)$

$\lim_{x\to 1^-} f(x)$ $\qquad$ $\lim_{x\to 1^+} f(x)$ $\qquad$ $\lim_{x\to 1} f(x)$ $\qquad$ $f(1)$

Note: $\lim_{x\to c} f(x) = L$ (that is, it exists) if and only if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \text{L}$

Note that with limits, we only care about the behavior of the function *near* the given x-value, not *at* the given x-value. So, in all three graphs below, $\lim_{x\to 4} f(x) = 3$.



Numerically

Consider $f(x) = \frac{x^2-1}{x-1}$. What is $f(1)$? ________________

Now what happens to $f(x)$ as x gets close to 1?

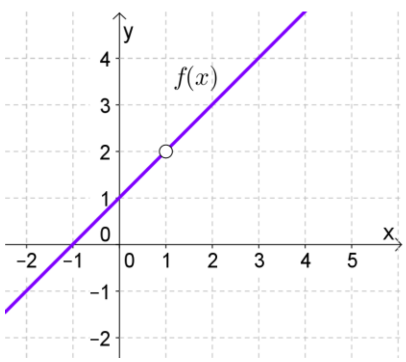
x	0.8	0.99	0.999	1	1.001	1.01	1.2
$f(x)$							

It looks like $f(x)$ approaches _____ as x approaches 1.

Using our limit notation, we can write:

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

And here's what the picture of $f(x) = \frac{x^2-1}{x-1}$ looks like:

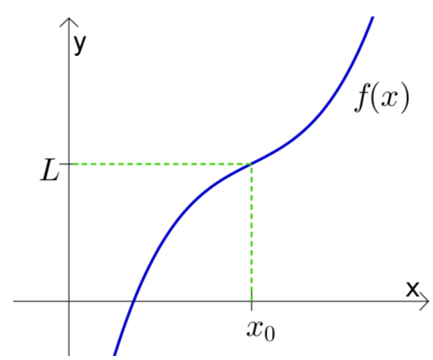


A rough definition of the limit:

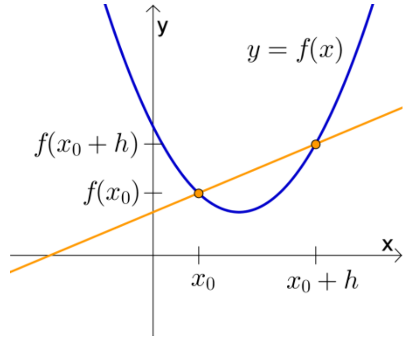
If $f(x)$ can be as close to L as we like by choosing x-values close to a number x_0 (from both sides), then L is the **limit of $f(x)$ as x approaches x_0**. That is,

$$\lim_{x\to x_0} f(x) = L$$

(read: "the limit of $f(x)$ as x approaches x_0 is L")

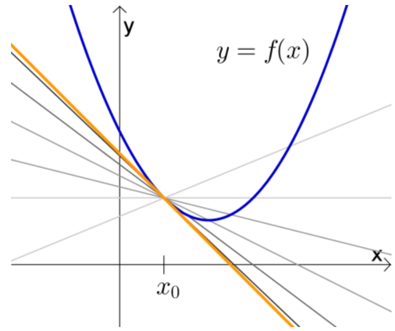


In general, the slope of the secant line over the interval $[x_0, x_0 + h]$ is:



$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + h) - f(x_0)}{(x_0 + h) - x_0} = \frac{f(x_0 + h) - f(x_0)}{h}$$

We can find the slope of the tangent line at x_0 by looking at secant slopes as h (our interval width) approached 0. With limits, we write the tangent slope like this:



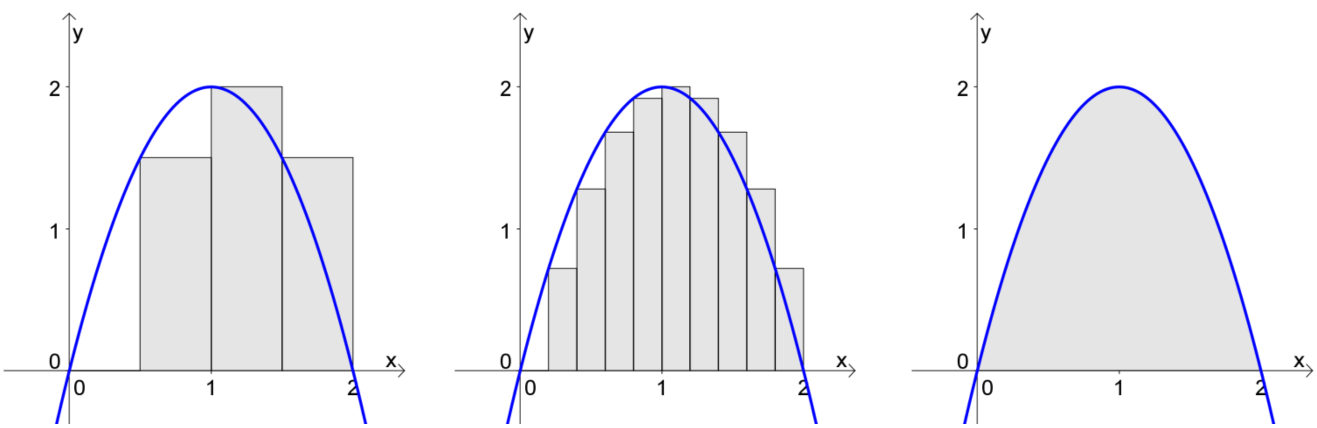
$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

The name given to this particular limit is the **derivative** of f at the point x_0. It is written $f'(x_0)$.

The derivative pops up across many fields of study. Here are some examples:
Velocity is the derivative of displacement.
Marginal profit is the derivative of profit.
Growth rate is the derivative of population.

In general, if you want to know the rate at which a function changes, then you want its derivative.

Another important calculus concept defined in terms of limits is the **integral**. Here's a brief glimpse:



$$\sum_{k=1}^{n} f(c_k)\Delta x \xrightarrow[n \to \infty]{} \int_0^2 f(x)\,dx$$

Q: What are the next two letters in the following sequence and why?
W A T N T L I T F S _ _